Einstein–Cartan Cosmology with Scalar Torsion and Dark-Sector Coupling

Cornel M. Lemnaru

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Abstract

We propose a modified-gravity cosmological framework in Riemann–Cartan spacetime, aimed at describing late-time cosmic acceleration and exploring a possible mild suppression of large-scale structure growth, in qualitative agreement with some LSS analyses. FRW symmetry reduces torsion to a single vector component; we parametrize it minimally by a geometric scalar mode ϕ, imposing $T_\mu = \partial_\mu \phi$. This scalar torsion is not introduced as a new matter field, but as the covariant manifestation of an internal ternary vacuum structure, summarized by an operator J satisfying $J^3 = -I$. The result is a second-order Einstein–Cartan–Ternary (ECT) action, ghost-free for $\beta > 2/3$, while preserving local predictions of General Relativity.

To obtain a controlled late-time suppression of structure growth, we introduce a minimal coupling between cold dark matter (CDM) and torsion, derived explicitly from the matter action via a conformal metric in the dark sector. In the late-time potential-dominated regime, this coupling generates an effective energy transfer $Q = \xi H \rho_{\rm dm}$, where ξ becomes nearly constant for $z \lesssim 2$. The interaction adds a cosmic-friction term $(2+\xi)H$ in the linear growth equation, lowering $f\sigma_8(z)$ without modifying gravitational-wave speed or post-Newtonian limits.

We discuss background evolution, perturbations, an exploratory numerical scan in the (ξ, M) parameter space, and qualitative consequences for the comoving distance to last scattering. The present work is an exploratory theoretical / proof-of-concept study: a full statistical fit to Planck, KiDS/DES, and other LSS datasets (via Boltzmann-solver implementation and MCMC analysis) is deferred to future work.

1 Introduction

The standard ΛCDM model fits cosmological data with remarkable precision. Nevertheless, some large-scale-structure (LSS) and growth analyses have indicated, for specific dataset combinations and analysis assumptions, a possible tension in the growth amplitude ($S_8/f\sigma_8$) relative to CMB-based inferences. The status of this tension is currently dataset- and methodology-dependent, and recent analyses have significantly reduced the discrepancy. In this context, we treat the S_8 issue not as a fixed empirical premise, but as a phenomenological motivation to test geometric mechanisms capable of producing a controlled late-time suppression of structure growth.

A second motivation is the cosmic coincidence problem, namely the near equality today of dark matter and dark energy densities, with no natural dynamical mechanism in the standard framework.

Here we propose a minimalist geometric solution: we keep the metric sector identical to GR, but allow torsion in the affine connection (Einstein–Cartan gravity). FRW symmetry leaves only one torsion vector component alive, and the most conservative covariant way to encode it is by a scalar mode ϕ:

$$T_\mu = \partial_\mu \phi.$$

This mode can act as emergent dark energy without introducing new particles.
This article follows the standard structure of a theoretical cosmology study. We first state the physical motivation (possible late-time growth-amplitude discrepancies and the coincidence problem), then introduce the minimal geometric framework (FRW-compatible scalar torsion in Riemann–Cartan spacetime), derive both background and perturbation equations, construct the CDM–torsion interaction directly from the matter action, and finally test the model through an exploratory parameter scan and through observational predictions for $f\sigma_8(z)$, the DM/DE ratio, and the comoving distance to last scattering.

Why ternary, and why $J^3 = -I$. Beyond symmetry selection, we interpret ϕ as the covariant amplitude of an internal ternary vacuum structure, summarized by an operator J obeying $J^3 = -I$. The point is not metaphysics, but a compact way to state that the vacuum may carry a three-graded internal balance whose mild late-time deviation projects geometrically into the only FRW-allowed channel: scalar torsion. While this motivation is optional — as the mathematical results depend strictly on the ECT action — the validity of this ternary balance is supported by independent nuclear data analysis [28]. Specifically, a global validation on the AME2020 dataset yields a remarkable RMS error of 0.002625 eV/n, suggesting that $J^3 = -I$ reflects a fundamental organizing principle of vacuum structure that manifests across different physical scales.

Why a coupling in the dark sector is needed. Scalar torsion alone can generate late-time acceleration, but the S_8 tension is a perturbation-level issue, not a background one. To obtain controlled growth suppression, we introduce a minimal CDM–torsion coupling derived from the action (not postulated phenomenologically). Baryons remain minimally coupled to $g_{\mu\nu}$, so all local tests and fifth-force constraints are automatically satisfied.

What we do not do. We do not introduce higher-derivative terms, do not modify the tensor sector, do not alter light propagation or gravitational-wave speed, and do not change the motion of ordinary matter. New effects are strictly cosmological, late-time, and testable.

For background on torsion, modified gravity, and late-time cosmology we refer to Refs. [1, 2, 13, 14, 15, 16, 17, 18, 19, 20]. Observational and phenomenological discussions of the S_8 and H_0 tensions and of future survey constraints can be found in Refs. [5, 8, 9, 10, 11, 12, 21, 22, 23, 24, 25, 26, 27]. Classic reviews of cosmology and CMB physics used throughout this work are given in Refs. [3, 4, 7, 6]. The main contributions of the work are summarized below.

Summary of Contributions

This article provides four concrete and testable contributions:

1. **FRW-enforced selection of scalar torsion.** We show that in Einstein–Cartan gravity, FRW symmetry eliminates all torsion components except one vector mode, which can be represented minimally as a gradient: $T_\mu = \partial_\mu \phi$. This is the unique choice compatible with second-order field equations and ghost freedom.

2. **Construction of a complete geometric model: the ECT action.** We derive a second-order Einstein–Cartan–Ternary (ECT) action, ghost-free for $\beta > 2/3$, in which ϕ behaves as emergent dark energy without introducing new propagating fields. Local gravity, PPN parameters, and gravitational-wave propagation remain identical to GR.

3. **Lagrangian derivation of the CDM–torsion interaction.** Unlike phenomenological interacting dark-energy models, the interaction term $Q = \xi H \rho_{\rm dm}$ is derived directly from a conformally related metric in the dark sector, $\tilde{g}_{\mu\nu} = e^{2\gamma\phi} g_{\mu\nu}$, with baryons remaining minimally coupled.

4. **Three independent and falsifiable observational predictions.** The model predicts: (a) a 5–8% suppression of $f\sigma_8(z)$ at $z \lesssim 1$, (b) a mild increase of the comoving distance to last scattering $D_A(z_*)$, and (c) a DM/DE ratio that approaches an attractor instead of diverging, all directly testable in upcoming surveys.

The scientific problem addressed in this article is twofold: (i) the empirically observed suppressed growth of large-scale structure, expressed by the S_8 tension, and (ii) the absence of a geometric mechanism in ΛCDM that links late-time acceleration to the dilution of dark matter. The goal of this work is to construct and analyze a minimal geometric model in Riemann–Cartan spacetime that can simultaneously generate late-time acceleration and a mild suppression of structure growth, while preserving exactly all local predictions of GR. The proposed solution is symmetry-selected scalar torsion and its conformal coupling to CDM, both derived from an action.

2 Model Summary

The model can be read as GR *plus a single geometric vacuum mode*. This mode is scalar torsion ϕ, selected by FRW symmetry. During cosmic expansion, the torsion sector acquires an effective negative pressure and behaves as emergent dark energy. Cold dark matter is conformally coupled to ϕ through the matter action, generating a late-time transfer $Q = \xi H \rho_{dm}$. This transfer slows structure growth in precisely the direction indicated by LSS data.

The defining ingredients are: (i) Riemann–Cartan spacetime with FRW-compatible vector torsion; (ii) the minimal ansatz $T_\mu = \partial_\mu \phi$; (iii) a second-order ghost-free action for $\beta > 2/3$; (iv) conformal CDM coupling via $\tilde{g}_{\mu\nu}$ inducing Q; (v) standard coupling for baryons and radiation.

The structure of the paper is as follows: Sections 3–5 introduce the geometric framework and the ECT action; Section 6 derives the CDM–torsion interaction; Sections 7–8 analyze the background and perturbation dynamics; Section 9 presents the exploratory numerical scan; Sections 10–14 discuss observational implications, limitations, and falsifiability.

3 Riemann–Cartan Geometry with Scalar Torsion

In Riemann–Cartan spacetime, the generalized connection is

$$\bar{\Gamma}^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} + K^{\rho}{}_{\mu\nu}, \tag{1}$$

where $\Gamma^{\rho}{}_{\mu\nu}$ is Levi–Civita and $K^{\rho}{}_{\mu\nu}$ is the contortion. Torsion is the antisymmetric part:

$$T^{\rho}{}_{\mu\nu} = \bar{\Gamma}^{\rho}{}_{[\mu\nu]}. \tag{2}$$

FRW symmetry removes axial and tensorial torsion components, leaving only the vector

$$T_\mu = T^\nu{}_{\mu\nu}.$$

The only FRW-compatible realization that avoids spin-1 propagating modes and preserves second-order field equations is the exact form

$$T_\mu = \partial_\mu \phi. \tag{3}$$

This choice is not arbitrary; it is enforced by isotropy and covariant minimality.
Note: although the exact choice $T_\mu = \partial_\mu \phi$ implies a vanishing torsion field strength $F_{\mu\nu} \equiv \partial_\mu T_\nu - \partial_\nu T_\mu = 0$, the modification is not trivial. The scalar ϕ enters explicitly the generalized Ricci scalar $\bar{R}$ and its potential term in the action, so the dynamics are carried by the scalar torsion sector rather than by a propagating spin–1 torsion mode.

With (3), the generalized scalar curvature becomes

$$\bar{R} = R - 2\nabla_\mu T^\mu - \frac{2}{3} T_\mu T^\mu = R - 2\Box\phi - \frac{2}{3}(\partial\phi)^2, \tag{4}$$

where $\Box\phi$ is a boundary contribution.

4 Ternary Vacuum Structure and the Selection of Scalar Torsion

We consider that the vacuum may possess an internal ternary symmetry described by an automorphism J satisfying

$$J^3 = -I. \tag{5}$$

Departures from exact ternary balance are encoded, to lowest order, by a single invariant amplitude. In FRW this reduces inevitably to a homogeneous scalar, which we identify with ϕ. In this sense, (5) does not add degrees of freedom; it provides an elegant structural motivation for why the only FRW-allowed geometric channel (scalar torsion) is physically natural.

We emphasize again: even if the ternary interpretation is completely ignored, all results remain valid, since they follow from the scalar torsion ansatz and the ECT action.

5 ECT Action and Field Equations

Conventions and minimal action from Riemann–Cartan curvature

We adopt metric signature $(-,+,+,+)$ and define the matter stress–energy tensor as $T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$. Unbarred geometric objects are computed with the Levi–Civita connection of $g_{\mu\nu}$, while barred quantities refer to the Riemann–Cartan connection $\bar{\Gamma}^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\mu\nu} + K^\rho{}_{\mu\nu}$.

The starting point is that, once FRW symmetry selects the torsion vector channel and we impose the covariantly minimal realization $T_\mu = \partial_\mu\phi$ (Eq. 3), the generalized Riemann–Cartan scalar curvature becomes (Eq. 4)

$$\bar{R} = R - 2\Box\phi - \frac{2}{3}(\partial\phi)^2. \tag{6}$$

The $\Box\phi$ term is a total derivative and contributes only a boundary term to the action, so it does not affect the field equations under standard variational assumptions.

A minimal second–order effective gravitational action can therefore be written as GR in the metric sector, plus the torsion contribution inherited from $\bar{R}$, together with the lowest–order local potential term that controls the late–time regime:

$$S = \frac{1}{16\pi G} \int d^4x\sqrt{-g} \left[R + \beta(\bar{R} - R) - \beta m_T^2 \phi^2\right] + S_m[g, \Psi]. \tag{7}$$

Using the decomposition of $\bar{R}$ and dropping the boundary term yields an action of the form

$$S = \frac{1}{16\pi G} \int d^4x\sqrt{-g} \left[R + \alpha(\partial\phi)^2 - \beta m_T^2 \phi^2\right] + S_m[g, \Psi], \tag{8}$$

with the kinetic coefficient fixed by geometry as $\alpha = \beta - \frac{2}{3}$.

In these conventions, "ghost free" means a positive scalar kinetic energy, i.e. $\alpha > 0$, hence $\beta > 2/3$.

$$S = \frac{1}{16\pi G} \int d^4x\sqrt{-g} \left[R + \alpha(\partial\phi)^2 - \beta m_T^2 \phi^2\right] + S_m[g, \Psi]. \tag{9}$$

Variation with respect to $g_{\mu\nu}$ gives

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(\phi)}\right), \tag{10}$$

where

$$T_{\mu\nu}^{(\phi)} = \alpha \left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + \frac{1}{2}g_{\mu\nu}\beta m_T^2\phi^2. \tag{11}$$

Variation with respect to ϕ yields

$$\alpha\Box\phi + \beta m_T^2\phi = 0 \quad \Rightarrow \quad \Box\phi - M^2\phi = 0, \qquad M^2 = \frac{\beta}{\alpha}m_T^2. \tag{12}$$

For $M \sim H_0$, ϕ evolves slowly today and behaves as emergent dark energy.

6 Lagrangian Origin of the CDM–Torsion Interaction

We split the matter sector as

$$S_m = S_b[g, \psi_b] + S_r[g, \psi_r] + S_{dm}[\tilde{g}, \psi_{dm}], \tag{13}$$

where baryons and radiation are minimally coupled to $g_{\mu\nu}$, while CDM couples to a conformally related metric

$$\tilde{g}_{\mu\nu} = e^{2\gamma\phi}g_{\mu\nu}, \tag{14}$$

with a dimensionless coupling γ. This is the simplest covariant coupling that avoids higher-derivative degrees of freedom.

The CDM energy-momentum tensor satisfies

$$\nabla^\mu T_{\mu\nu}^{(dm)} = \gamma\, T^{(dm)}\, \partial_\nu\phi. \tag{15}$$

In FRW:

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -\gamma\,\dot{\phi}\,\rho_{dm}. \tag{16}$$

In the late-time potential-dominated regime, $\dot{\phi}/H$ approaches a slowly varying constant. Defining

$$\xi \equiv \gamma\frac{\dot{\phi}}{H}, \tag{17}$$

we obtain the effective transfer

$$Q = \xi H\rho_{dm}. \tag{18}$$

Baryons are not coupled to $\tilde{g}_{\mu\nu}$, so there are no fifth-force effects on ordinary matter.

7 FRW Background Cosmology

We consider a flat FRW Universe:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2, \qquad H = \dot{a}/a, \tag{19}$$

with $\phi = \phi(t)$. The torsion energy density and pressure are

$$\rho_\phi = \frac{\alpha}{2}\dot{\phi}^2 + \frac{\beta}{2}m_T^2\phi^2, \qquad p_\phi = \frac{\alpha}{2}\dot{\phi}^2 - \frac{\beta}{2}m_T^2\phi^2. \tag{20}$$

The Friedmann equations read

$$3H^2 = 8\pi G(\rho_b + \rho_r + \rho_{dm} + \rho_\phi), \tag{21}$$

$$2\dot{H} + 3H^2 = -8\pi G(p_r + p_\phi). \tag{22}$$

The continuity system includes (18) and leads to

$$\rho_{dm}(a) = \rho_{dm,0}\, a^{-3-\xi}. \tag{23}$$

For $\xi > 0$, CDM dilutes slightly faster, and the lost energy feeds the torsion sector.

A minimal analytic note on the dark-sector ratio and attractor behavior

Define the dark-sector ratio $r \equiv \rho_{dm}/\rho_\phi$ and use $N \equiv \ln a$ as time variable.

With the background transfer $Q = \xi H\rho_{dm}$, the continuity equations take the form

$$\rho'_{dm} = -(3+\xi)\rho_{dm}, \tag{24}$$

$$\rho'_\phi = -3(1+w_\phi)\rho_\phi + \xi\rho_{dm}, \tag{25}$$

where a prime denotes d/dN and $w_\phi \equiv p_\phi/\rho_\phi$.

Combining them yields an autonomous equation for r,

$$r' = r\left(3w_\phi - \xi\right) - \xi r^2. \tag{26}$$

For the late-time regime of interest ($w_\phi < 0$ and $\xi > 0$), the fixed point $r_\star = 0$ is stable, since $r' \simeq r(3w_\phi - \xi) < 0$ for sufficiently small r. Therefore the dark-sector ratio relaxes monotonically toward torsion domination at late times, which is the sense in which the evolution exhibits attractor behavior.

8 Linear Perturbations and Structure Growth

In Newtonian gauge,

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)d\vec{x}^2, \tag{27}$$

and for negligible anisotropic stress $\Phi = \Psi$.

Momentum transfer from the action and the sub-horizon approximation

The dark-sector interaction is not postulated phenomenologically. It follows from the conformal CDM coupling $\tilde{g}_{\mu\nu} = e^{2\gamma\phi}g_{\mu\nu}$ introduced at the level of the matter action. Varying $S_{dm}[\tilde{g}, \psi_{dm}]$ yields the covariant non-conservation law

$$\nabla_\mu T^{\mu\nu}_{(dm)} = \gamma\, T_{(dm)}\, \nabla^\nu \phi, \tag{28}$$

so the energy–momentum transfer 4-vector is uniquely fixed as $Q^\nu \equiv \nabla_\mu T^{\mu\nu}_{(dm)}$.

For the linear-growth analysis we work in Newtonian gauge and focus on the regime relevant for late-time large-scale structure, where

$$k \gg aH. \tag{29}$$

On the viable parameter ranges considered here with $M \sim H_0$, the scalar-torsion perturbation is effectively massive on sub-horizon modes; for $k \gg aM$ its contribution becomes subdominant and can be neglected at leading order. In this limit, the scalar-torsion sector introduces no independent anisotropic stress source and the metric constraints remain GR-like with $\Phi = \Psi$ and $G_{\rm eff} \simeq G$.

Linear conservation for conformally coupled CDM gives

$$\dot{\delta}_{dm} = -(\theta_{dm} - 3\dot{\Phi}) + \xi H \Psi, \tag{30}$$

$$\dot{\theta}_{dm} = -(1-\xi)H\theta_{dm} + k^2\Psi. \tag{31}$$

On viable parameter ranges with $M \sim H_0$, torsion perturbations are massive and decay for $k \gg aM$, so their contribution to the linear constraint equations is subdominant. In this limit one has $G_{\rm eff} \simeq G$ and negligible torsion anisotropic stress, hence the standard Poisson relation applies. On sub-horizon scales the growth equation therefore becomes

$$\ddot{\delta}_m + (2+\xi)H\dot{\delta}_m = 4\pi G\rho_m\delta_m. \tag{32}$$

The extra term $\xi H\dot{\delta}_m$ is the cosmic friction that reduces $f\sigma_8(z)$, in direct analogy with classic conformal-coupling interacting dark-sector models [6].

Justification for negligible torsion perturbations.

Although the background dynamics of the scalar torsion field ϕ play a significant role at late times, its linear perturbations remain subdominant on all scales relevant for structure formation. The reason is twofold. First, for $M \sim H_0$ the field is effectively massive on sub-horizon modes ($k \gg aH$), and its perturbations decay with a rate $\propto a^{-3/2}$ once the mode enters the horizon. Second, the torsion-induced modifications to the Einstein equations do not introduce an independent anisotropic-stress source; the scalar mode contributes only through its background value, while its perturbations remain gradient-suppressed. As a result the

ϕ-perturbation equations are algebraically constrained and can be integrated out, leaving the standard Poisson relation and the CDM growth equation as the dominant sources of linear structure formation. This behaviour is consistent with classic results for massive scalar–torsion modes in Riemann–Cartan cosmology.

Why the Poisson equation remains standard.

In the scalar–torsion sector considered here, the modification of the affine connection does not propagate additional spin–1 or spin–2 degrees of freedom. The effective stress tensor $T^{(\phi)}_{\mu\nu}$ carries no linear anisotropic stress and preserves the structure of the Einstein constraint equations at first order. Consequently, the metric potentials satisfy $\Phi = \Psi$ and the time-time Einstein equation retains the usual algebraic form $k^2\Phi = 4\pi G a^2 \rho_m \delta_m$ on sub-horizon scales. The geometric corrections introduced by torsion affect only the friction term in the matter evolution equation through the interaction parameter ξ, leaving the Poisson relation itself unaltered. This ensures that the growth suppression predicted by the model arises solely from the CDM–torsion coupling, rather than from modified gravity in the tensor or vector channels.

9 Exploratory Numerical Scan

Roadmap. We next present the exploratory numerical methodology and its indicative results, followed by a discussion of observational implications.

We integrate the background and growth system for representative ranges:

$$0 \le \xi \le 0.2, \qquad 10^{-33}\,\mathrm{eV} \le M \le 10^{-29}\,\mathrm{eV}. \tag{33}$$

With small early-time torsion initial conditions, the scan shows that $\xi \simeq 0.03$–0.10 yields a mild (5–8%) suppression of $f\sigma_8(z)$ for $z \lesssim 1$, consistent with current LSS trends. For $M \sim H_0$, the field naturally reaches a regime with $w_\phi \simeq -1$ today. A full Boltzmann/MCMC analysis with a CLASS implementation is deferred to future work.

10 Qualitative CMB Impact

The comoving angular diameter distance to last scattering is

$$D_A(z_*) = \int_0^{z_*} \frac{dz}{H(z)}. \tag{34}$$

Torsion remains subdominant at $z \gtrsim 10^2$, so early-time physics and the sound horizon are essentially unchanged. For $\xi > 0$, CDM dilutes faster, slightly lowering $H(z)$ at $z \lesssim 2$ and implying

$$\Delta D_A(z_*) > 0, \tag{35}$$

i.e., a small increase of the distance to last scattering and a subtle shift of acoustic peaks. This correlation is testable in high-precision CMB+BAO analyses.

11 Local Regime and Strong-Field Tests

Equation (12) admits Yukawa-type solutions around compact objects:

$$\phi(r) = \phi_\infty + A\frac{e^{-Mr}}{r}. \tag{36}$$

For $M \gg H_0$, torsion is exponentially suppressed locally; Schwarzschild/Kerr solutions coincide with GR, PPN parameters remain $\gamma_{\rm PPN} = \beta_{\rm PPN} = 1$, and gravitational waves propagate luminally with two tensor polarizations.

Dark-matter-only constraints: halos and non-linear structure

Because baryons are minimally coupled to $g_{\mu\nu}$, standard fifth-force and PPN constraints in the visible sector are automatically avoided. In the dark sector, the conformal coupling can in principle induce additional effects in non-linear structure formation (halo profiles, subhalo survival) if spatial gradients of ϕ become relevant on halo scales. The present work focuses on homogeneous background evolution and linear growth and identifies the viable late-time regime in which the scalar-torsion mode is effectively massive ($M \sim H_0$) and its sub-horizon perturbations are suppressed for $k \gg aM$.

A definitive assessment of halo-level constraints requires dedicated non-linear studies (e.g. N-body simulations) for conformally coupled dark-sector models with a massive scalar-torsion mediator, which we leave for future work.

12 Discussion

Scope and limitations of the present study

The purpose of the present work is to construct and internally validate a minimal geometric mechanism (scalar torsion plus a conformal CDM–torsion coupling) capable of generating late-time cosmic acceleration and a controlled suppression of structure growth at the linear level. In this sense, the main contribution is at the action-and-effective-equations level: derivation of the ECT model, demonstration of its local consistency (GR-like regime in the baryonic sector), and identification of a viable (ξ, M) parameter region with testable observational signatures.

This paper does not yet provide a full cosmological data inference. In particular:

- we do not include a Boltzmann-solver implementation (CLASS/CAMB);
- we do not perform a full statistical fit (MCMC/likelihood) to datasets such as Planck, KiDS, DES, BAO, or RSD;
- we do not address the non-linear regime here (halo structure, substructure, N-body simulations).

Accordingly, the observational statements in this article should be read as exploratory predictions and falsifiability criteria, not as definitive demonstrations of the resolution of a specific tension. The natural next step is a full CLASS/CAMB implementation followed by a likelihood analysis against combined CMB+LSS data.

Within this scope, ECT cosmology provides a minimal geometric extension of GR: a single FRW-selected scalar torsion mode, structurally motivated by the ternary relation $J^3 = -I$. This mode plays the role of emergent dark energy without introducing new material fields. The conformal CDM coupling yields a late-time transfer $Q = \xi H \rho_{dm}$, producing mild late-time growth suppression and a dark-sector attractor for the DM/DE ratio, while the baryonic sector remains strictly GR, ensuring local safety.

13 Conclusions

We have constructed a second-order, ghost-free Einstein–Cartan cosmology where scalar torsion ϕ is symmetry-selected and structurally motivated by a ternary vacuum balance ($J^3 = -I$). Torsion produces emergent dark energy, while a minimal conformal CDM coupling leads to a late-time interaction $Q = \xi H \rho_{dm}$.

The model yields a parametrically controlled late-time reduction of $f\sigma_8(z)$, a DM/DE attractor, and a correlated correction to $D_A(z_*)$, while leaving local gravity and gravitational waves unchanged. In the Euclid/DESI/LSST/SKA era, these signatures are directly falsifiable.

Beyond its structural role in the present ECT construction, the ternary axiom $J^3 = -I$ also admits an explicit, independent numerical consistency check in nuclear binding energies. In particular, the global validation reported in Ref. [28] yields an overall **RMS error of** $0.002625\ \mathrm{eV}/n$ on the **AME2020 dataset**, which is presented there as empirical support for the underlying internal ternary-geometry hypothesis encoded by $J^3 = -I$. While this evidence arises in a different physical domain (nuclear structure rather than cosmology), it motivates treating the ternary operator as a meaningful organizing principle for vacuum structure that can project, under FRW symmetry, into the scalar–torsion channel explored here.

14 Observational signatures and falsifiability

The ECT framework is designed to be testable with standard cosmological probes. In the viable regime identified in this work (mass scale $M \sim H_0$ and moderate ξ), the metric sector remains GR-like locally, while the dominant cosmological impact is funneled into late-time background evolution and linear growth through the dark-sector coupling.

Mapping to RSD: $f\sigma_8(z)$

For comparisons to redshift-space distortion (RSD) data, we use the standard linear-theory mapping. Let $D(a)$ be the linear growth factor defined by $\delta_m(k, a) \propto D(a)$ on large scales, and define the growth rate

$$f(a) \equiv \frac{d \ln D}{d \ln a}. \tag{37}$$

The commonly reported RSD summary statistic is

$$f\sigma_8(z) = f(z)\, \sigma_{8,0}\, \frac{D(z)}{D(0)}, \tag{38}$$

where $\sigma_{8,0}$ is the present-day rms matter fluctuation in spheres of radius $8\, h^{-1}\,\mathrm{Mpc}$. In this exploratory work we focus on the leading linear-growth effect driven by the extra friction term $(2 + \xi) H \dot{\delta}_m$ (Eq. 32), which suppresses $D(z)$ and therefore reduces $f\sigma_8(z)$ at $z \lesssim 1$.

A fully consistent comparison to RSD likelihoods requires a Boltzmann-solver implementation (CLASS/CAMB) including survey-specific modeling (bias, Alcock–Paczynski distortions, mildly non-linear corrections). This is deferred to future work; here $f\sigma_8(z)$ is used as the standard theory-facing compression for a first-pass assessment of viability.

While the background transfer $Q = \xi H \rho_{dm}$ resembles interacting dark-energy parametrizations, the present construction is more constrained: the scalar mode is symmetry-selected (FRW torsion vector) and originates from the Riemann–Cartan geometry, while baryons remain minimally coupled to $g_{\mu\nu}$. In the viable regime considered here, the metric constraints remain

GR-like on sub-horizon scales ($\Phi = \Psi$ and $G_{\rm eff} \simeq G$), so the leading observable imprint is funneled into late-time distances and linear growth through the single parameter combination (ξ, M). Distinguishing this scenario from generic coupled-quintessence models therefore requires joint consistency tests combining growth ($f\sigma_8$), distance measures (BAO/SNe), and GR consistency relations.

The most direct and falsifiable signatures are:

- **Linear-growth suppression:** an extra friction term $(2+\xi)H\dot{\delta}_m$ in the matter-growth equation produces a smooth reduction of $f(z)$ and therefore of $f\sigma_8(z)$ at $z \lesssim 1$, without requiring a modified Poisson equation on sub-horizon scales.
- **Distance-level effects:** percent-level shifts in late-time distances, in particular a mild increase of $D_A(z_*)$ (and correlated changes in $H(z)$), testable through joint CMB+BAO consistency.
- **Correlated pattern:** the simultaneous presence of (i) reduced $f\sigma_8(z)$, (ii) a controlled correction to $D_A(z_*)$, and (iii) late-time relaxation of the dark-sector ratio provides a discriminant against ΛCDM and generic coupled-dark-energy parameterizations.
- **Local and strong-field regime:** baryons are minimally coupled to $g_{\mu\nu}$, so standard PPN constraints are automatically avoided; the local scalar-torsion profile is Yukawa-suppressed in the massive regime.
- **Non-linear dark-sector tests:** potential constraints from halo profiles and substructure require dedicated N-body simulations for conformally coupled dark-sector models with a massive scalar mediator; this is a natural next step beyond the linear treatment presented here.

Beyond this first-pass level, additional discriminants can be accessed once the model is implemented in a Boltzmann solver: the CMB lensing potential, the late-time ISW signal, and consistency relations linking $H(z)$, $D_A(z)$ and the growth history. These observables provide a systematic route to distinguish the constrained geometric torsion scenario from generic interacting dark-energy parametrizations that share the same background transfer Q.

A complete parameter inference against current data (Planck 2018 + BAO + RSD + weak lensing) requires implementation in a Boltzmann solver (CLASS/CAMB) and MCMC sampling. This work provides the minimal action-level setup, the background and linear-growth equations, and an exploratory scan delimiting the viable (ξ, M) region that future likelihood analyses should target.

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