Unified Vacuum Dynamics
from a Ternary Internal Symmetry

Cornel Lemnaru

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Abstract

We propose a unified vacuum architecture in which gravity and electrodynamics emerge as complementary projections of an internal structure endowed with ternary symmetry. The foundation of the model is an internal operator J satisfying the closure relation $J^3 = -I$, which enforces a spectral tripartition of the vacuum and yields an effective Lagrangian composed of three correlated sectors: (i) a metric sector associated with gravity, (ii) a photonic sector interpreted as a ternary vacuum mode, and (iii) a neutrino sector that stabilizes the vacuum energy through a structural cancellation mechanism.

In this construction, the cosmological constant is not a tuned parameter but the emergent result of internal phase summation, while torsion appears as a natural degree of freedom with relevance at cosmological scales. The ternary structure further ensures coherent propagation between gravitational and photonic modes in dynamical vacuum regimes.

The model produces testable phenomenological predictions, including a controlled variation of the fine-structure constant $\alpha(z)$, conditions for the equality of the speeds of light and gravitational waves, corrections to light deflection in strong gravitational fields, and distinctive signatures in gravitational-wave propagation through a ternary-polarized vacuum. We compare the results with the standard GR+QFT framework and formulate explicit falsifiability criteria, identifying observational domains where deviations from the current paradigm become accessible to modern instrumentation.

The proposed ternary architecture thus provides a coherent and verifiable scenario in which the vacuum behaves as a dynamic medium with a discrete internal symmetry, naturally supporting the unification of fundamental interactions.

1 Introduction

Fundamental physics has succeeded in unifying—remarkably precisely—two profoundly different theoretical frameworks: quantum field theory (QFT), which describes interactions at microscopic scales, and general relativity (GR), which describes gravitation and the structure of spacetime at macroscopic and cosmological scales. Together, these theories reproduce with great accuracy an exceptionally broad range of phenomena, from LHC collisions to the large-scale evolution of the Universe. Yet conceptually, their combined architecture remains surprisingly fragile.

Four major tensions stand out:

1. **The vacuum energy problem.** QFT estimates of vacuum energy lead, even under conservative assumptions, to an energy density roughly 10^{120} times larger than the value inferred from cosmological observations. Reconciling this enormous discrepancy by fine-tuning a cosmological constant appears artificial and lacks a structural mechanism.

2. **The absence of torsion in GR.** Natural geometric extensions of general relativity (such as Riemann–Cartan theories) allow torsion, yet standard GR excludes it by construction, without a physical principle explaining why the connection should be exactly Levi–Civita. Moreover, when torsion is introduced, it typically appears as an ad hoc term, not as the inevitable consequence of a deeper symmetry.

3. **Possible variation of the fine-structure constant α.** There exist observational indications that α may exhibit temporal or spatial variations on cosmological scales. Standard models attempting to explain this phenomenon introduce additional scalar fields (dilaton- or moduli-like) with phenomenologically chosen potentials and couplings, lacking a unified structural justification.

4. **The structure of the neutrino sector.** The existence of three types of neutrinos, with pronounced oscillations and extremely small masses, is incorporated into the Standard Model through a phenomenologically introduced mixing matrix (PMNS). However, there is no structural explanation for why precisely three neutrino modes arise, nor for the particular values of the mixing angles and phases.

The fact that these four problems arise in such different contexts—theory of the vacuum, spacetime geometry, fundamental constants, and the neutrino sector—suggests the absence of a deep unifying principle. In this work, we propose such a principle in the form of a *ternary architecture* of the vacuum and fundamental interactions.

The central idea is the introduction of an internal operator J, acting in an internal space associated with the vacuum (or the symmetry sector of fields), which satisfies

$$J^3 = -I. \tag{1}$$

This cubic relation induces a ternary spectral structure with three distinct eigenvalues (the roots of $\lambda^3 + 1 = 0$) and a canonical decomposition into three quasi-orthogonal projections P_0, P_1, P_2. From this projective structure, the following emerge:

- a natural mechanism for **structural cancellation of vacuum energy**, based on interference among the three internal components;
- the appearance of **torsion** as an emergent phenomenon required for geometric compatibility with the ternary symmetry;
- an interpretation of the **variation of the fine-structure constant** $\alpha(z)$ as a geometric effect of the evolution of J and of the internal normalization of the electromagnetic sector;
- a **natural structuring of the neutrino sector** into three internal modes, providing a geometric framework for oscillations and for the CP phase.

Thus, phenomena that are treated separately in the standard paradigm—the vacuum-energy problem, possible variation of α, torsion, neutrino oscillations—become, in the ternary architecture, different manifestations of the same internal operator J and its triple projective decomposition.

Objectives of the work

The main goal of the present work is the formulation and detailed analysis of this ternary architecture, both mathematically and phenomenologically. More concretely, we aim to:

1. Construct the **complete mathematical formalism** of the ternary operator J: spectral properties, internal projections P_k, projective identities, and geometric interpretation.
2. Integrate the ternary symmetry into a **Riemann–Cartan geometric framework**, highlighting how the variation of J induces contorsion and emergent torsion.
3. Formulate a **complete Lagrangian** including the gravitational sector, the internal dynamics of J, projective terms, and matter couplings to the internal projections.
4. Derive the **equations of motion** for the vielbein, connection, operator J, and projections, demonstrating that the system is mathematically closed and consistent.
5. Demonstrate explicitly the **vacuum-energy cancellation mechanism** as the result of ternary interference, and discuss the stability conditions for this cancellation.
6. Deduce the **variation of** $\alpha(z)$ from the dynamics of J, and identify the magnitude of the effect relative to the sensitivity of current and upcoming instruments.
7. Propose a **structural modelling of the neutrino sector**, in which the three neutrino types correspond to internal projections P_k, and analyse the implications for the PMNS matrix.
8. Extract **concrete phenomenological predictions** for gravitational waves, cosmology, neutrinos, and the variation of fundamental constants, in the form of testable signatures.
9. Formulate **clear falsifiability criteria**, indicating the observational scenarios in which the ternary architecture is unambiguously ruled out.

Novelty and contributions

The main contributions of the work may be summarized as follows:

- We introduce and develop a **ternary mathematical framework** for the vacuum and interactions, based on an operator J satisfying $J^3 = -I$ and the associated internal projections.
- We show that this ternary symmetry leads to a **structural cancellation mechanism for vacuum energy**, robust against moderate perturbations and independent of the detailed form of the potential $\mathcal{V}(J)$.
- We demonstrate that **torsion** is not an optional term but a geometric necessity arising from covariant compatibility with the ternary symmetry, and we analyse its impact on gravitational-wave propagation.
- We deduce that the **variation of the fine-structure constant** $\alpha(z)$ can be interpreted as a geometric effect of the evolution of J, without introducing additional scalar fields.
- We propose a **structural interpretation of the neutrino sector**, in which the three neutrino modes correspond to internal projections P_k, and where the PMNS matrix and CP phase arise naturally from the ternary symmetry.

- We identify **distinctive phenomenological signatures**—longitudinal gravitational mode, gravitational birefringence, small variations of $\alpha(z)$, fine PMNS deviations, sub-percent modifications of $H(z)$—and provide a set of **experimental tests** capable of confirming or refuting the model.

Limitations and domain of validity

Although the ternary architecture offers a unified and predictive framework, it explicitly exhibits several limitations:

- The detailed form of the potential $\mathcal{V}(J)$ is not yet derived from a more fundamental principle; in this work we rely only on its general properties (minima satisfying $J^3 = -I$).
- The analysis of $\alpha(z)$ variation is presented at the level of order of magnitude and functional structure, rather than as a full numerical prediction for a single cosmological model.
- The neutrino sector is treated structurally, and a complete calculation of the neutrino mass spectrum within the ternary architecture is left for future work.
- We do not address in detail the couplings to each sector of the Standard Model; instead we focus on the phenomena where ternary symmetry yields distinctive signatures.

These limitations do not affect the conceptual core of the construction: the fact that a single internal object—the ternary operator J—can tie together, within a common framework, four challenging problems of modern physics. In the following sections, we develop in detail the mathematical formalism, geometric structure, complete Lagrangian, and phenomenological implications of this ternary architecture.

2 Mathematical foundations of the ternary operator J

The ternary architecture originates from a simple yet highly constraining mathematical object: an internal operator J satisfying the cubic identity

$$J^3 = -I. \tag{2}$$

This apparently modest relation is responsible for the entire theoretical edifice of the model. In this section we rigorously develop the mathematical properties of J, the structure of the internal space, the associated projections, projective identities, ternary periodicity, and their geometric meaning.

2.1 Ternary spectrum and the roots of the minimal polynomial

The identity (2) implies that J is an operator with minimal polynomial

$$p(\lambda) = \lambda^3 + 1,$$

whose roots are

$$\lambda_k = e^{i(\pi+2\pi k)/3}, \qquad k = 0, 1, 2,$$

that is, explicitly,

$$\lambda_0 = e^{i\pi/3}, \qquad \lambda_1 = e^{i\pi} = -1, \qquad \lambda_2 = e^{i5\pi/3}.$$

These are the three solutions of the spectral equation $\lambda^3 = -1$.

For convenience, we also introduce the cubic roots of unity:

$$\omega = e^{2\pi i/3}, \qquad \omega^3 = 1, \qquad 1 + \omega + \omega^2 = 0.$$

These phases $(1, \omega, \omega^2)$ appear in internal ternary combinations, although ω is not an eigenvalue of J, since $J^3 = -I$.

Thus, the spectrum of J comprises three distinct eigenvalues, generating a natural partition of the internal space into three quasi-orthogonal subspaces forming an equilateral-triangle structure analogous to the phases $(1, \omega, \omega^2)$.

This complex spectrum is essential not only for the internal tripartition but also for the interference effects leading later to the cancellation of vacuum energy.

2.2 Internal projections P_0, P_1, P_2

Using the standard construction of projectors for a diagonalizable operator, we define the projections associated with each root.

Writing explicitly the eigenvalues,

$$\lambda_k = e^{i(\pi+2\pi k)/3}, \qquad k = 0, 1, 2,$$

the spectral projectors may be expressed as polynomials of degree at most two in J:

$$P_k = \frac{1}{3}\Big(I + \lambda_k^{-1} J + \lambda_k^{-2} J^2\Big), \qquad k = 0, 1, 2. \tag{3}$$

They satisfy the fundamental identities:

$$P_k^2 = P_k, \qquad P_i P_j = 0 \text{ for } i \neq j, \qquad P_0 + P_1 + P_2 = I,$$

forming a complete decomposition of the identity.

A key physical property is that any internal field Ψ has a unique decomposition:

$$\Psi = P_0\Psi + P_1\Psi + P_2\Psi,$$

into three distinct internal modes with fixed relative phases.

2.3 Ternary periodicity and the internal cycle

The operator J acts cyclically on the projectors, implementing a three-step internal rotation:

$$JP_0 = P_1 J, \qquad JP_1 = P_2 J, \qquad JP_2 = P_0 J. \tag{4}$$

This induces the cycle:

$$\Psi_0 \to \Psi_1 \to \Psi_2 \to -\Psi_0,$$

where $\Psi_k = P_k \Psi$.

Ternary periodicity is the internal signature of the identity $J^3 = -I$. This cycle is the structural mechanism that will later explain:

- internal oscillations in the neutrino sector,
- interference leading to vacuum-energy cancellation,
- gauge normalization variation and, implicitly, $\alpha(z)$ variation,
- the structure of ternary torsion.

2.4 Matrix representations of J

Two physically relevant representations are:

Cyclic (canonical) representation:

$$J = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad J^3 = -I.$$

Diagonal representation:

$$J_{\text{diag}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},$$

with trivial projections:

$$P_0 = \mathrm{diag}(1,0,0),\ P_1 = \mathrm{diag}(0,1,0),\ P_2 = \mathrm{diag}(0,0,1).$$

2.5 Projective identities and structural relations

An essential identity is:

$$JP_k = \omega^k P_k J.$$

Two further identities play a central role:

1. Cubic compatibility relation:

$$J^2 X + JXJ + XJ^2 = 0$$

for any $X = \nabla J$.

2. Phase sum:

$$1 + \omega + \omega^2 = 0.$$

From this follows the structural cancellation of vacuum energy.

2.6 Geometric interpretation

The ternary internal space is not a physical extra dimension, but a phase structure living in the internal fiber of fields. It influences:

- gauge-field normalizations,
- the geometric connection,
- fermionic dynamics,
- gravitational-wave propagation.

2.7 Conclusion of the section

The ternary operator J introduces a three-component internal geometry with cyclic relations, and the associated projections provide a complete decomposition of any internal field. The identity $J^3 = -I$ is far stronger than an ordinary symmetry: it enforces torsion, cancels vacuum energy, generates variation of fundamental constants, and structures the neutrino sector.

All phenomenological results of the ternary architecture rest on the mathematical structure developed in this section.

3 Full differential formalism

Integrating ternary symmetry into a geometric framework requires a formalism capable of accommodating both the variation of the internal operator J and a spacetime structure with possible torsion. The natural setting is Riemann–Cartan geometry, where the vielbein and connection are independent dynamical fields, and torsion is allowed and driven by internal structure. In this section we construct the complete formalism: vielbein, connection, torsion, curvature, contorsion, and the covariant derivative of J.

3.1 Vielbein and metric

The geometric description begins with a vielbein

$$e^a = e^a_{\ \mu}\, dx^\mu,$$

relating spacetime indices $(\mu, \nu, \dots)$ and local Lorentz indices $(a, b, \dots)$. The metric is

$$g_{\mu\nu} = \eta_{ab} e^a_{\ \mu} e^b_{\ \nu},$$

with $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$. The vielbein is treated as a field independent from the connection.

3.2 General connection and torsion

The spin connection is a 1-form

$$\omega^{ab} = \omega^{ab}_{\ \ \mu} dx^\mu, \qquad \omega^{ab} = -\omega^{ba},$$

with torsion

$$T^a = de^a + \omega^a_{\ b} \wedge e^b.$$

In GR, torsion is set to zero. In the ternary architecture, such a restriction is impossible: compatibility with the internal symmetry J requires emergent torsion.

3.3 Curvature

The curvature is

$$R^{ab} = d\omega^{ab} + \omega^a_{\ c} \wedge \omega^{cb}.$$

3.4 Levi–Civita connection and contorsion

The full connection decomposes as

$$\omega^{ab} = \omega^{ab}_{\mathrm{LC}} + K^{ab},$$

where $\omega^{ab}_{\mathrm{LC}}$ is torsion-free and K^{ab} is the contorsion:

$$K_{abc} = \frac{1}{2}(T_{abc} - T_{bca} + T_{cab}).$$

3.5 Covariant derivative of the ternary operator

The covariant derivative is

$$\nabla_\mu J = \partial_\mu J + [\Omega_\mu, J],$$

where Ω_μ is the internal connection.

Compatibility of projections requires:

$$\nabla_\mu P_k = 0.$$

Using their definition gives the cubic identity:

$$J^2(\nabla_\mu J) + J(\nabla_\mu J)J + (\nabla_\mu J)J^2 = 0. \tag{5}$$

3.6 Consequences: torsion is unavoidable

Identity (5) implies:

- the Levi–Civita connection cannot satisfy compatibility with ternary symmetry;
- ∇J cannot vanish except in trivial cases;
- the full connection must contain contorsion;
- torsion is proportional to ∇J.

Thus,

$$T^a = K^a{}_b(J) \wedge e^b,$$

with contorsion determined by the variation of J.

3.7 Internal energy and momentum

For any field Ψ,

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2, \qquad \Psi_k = P_k\Psi,$$

the covariant derivative acts differently on each mode, producing fixed internal phases ω^k, a mechanism essential for vacuum-energy cancellation.

3.8 Extended geometric interpretation

External geometry (vielbein + connection) and internal geometry (J + projections) form an extended fiber structure. In this framework:

- torsion encodes internal variation of J;
- the internal connection affects gauge propagation;
- the internal sector contributes to curvature through projections;
- internal variations determine macroscopic physical properties;
- spacetime becomes an active medium sensitive to internal structure.

3.9 Conclusion of the section

The full differential formalism of the ternary architecture extends Riemann–Cartan geometry by imposing covariant compatibility with ternary symmetry. The operator J is not merely an internal element but a structural generator determining torsion, influencing curvature, and governing field dynamics. This formalism constitutes the geometric foundation on which the complete Lagrangian and equations of motion will be built.

4 Complete Lagrangian of the ternary architecture

After establishing the geometric background (vielbein, connection, torsion) and internal ternary structure (operator J and projections P_k), we can formulate the complete action of the theory. It includes the gravitational sector, the internal dynamics of the ternary operator J, projective terms enforcing ternary consistency, and matter couplings to the internal structure. The resulting Lagrangian is the simplest set of terms ensuring compatibility with the identity $J^3 = -I$ and with standard geometric symmetries.

4.1 General structure of the action

The total action is

$$S = \int \left(\mathcal{L}_{\text{grav}} + \mathcal{L}_J + \mathcal{L}_{\text{proj}} + \mathcal{L}_{\text{matter}}\right). \tag{6}$$

The four sectors are:

- $\mathcal{L}_{\text{grav}}$: gravitational part in Riemann–Cartan geometry;
- $\mathcal{L}_J$: internal dynamics of the ternary operator J;
- $\mathcal{L}_{\text{proj}}$: projective terms enforcing ternary structure;
- $\mathcal{L}_{\text{matter}}$: matter couplings to internal projections and external connections.

This structure preserves diffeomorphisms, local Lorentz symmetry, and the internal ternary symmetry $J \to UJU^{-1}$ with $U^3 = I$.

4.2 Gravitational sector: generalized Einstein–Cartan theory

The gravitational part is the classical Einstein–Cartan action:

$$\mathcal{L}_{\text{grav}} = \frac{1}{2\kappa}\,\epsilon_{abcd}\,e^a \wedge e^b \wedge R^{cd}, \tag{7}$$

where

$$R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}.$$

Torsion is not set to zero. Instead, it appears in the equations of motion due to variation of J, not as spin-matter in the standard Einstein–Cartan theory.

4.3 Internal dynamics of the ternary operator J

The ternary operator is a dynamical internal field. The natural kinetic term is:

$$\mathcal{L}_J = \alpha_0\,\mathrm{Tr}((\nabla J) \wedge *(\nabla J))\,, \tag{8}$$

with potential $\mathcal{V}(J)$ having minima satisfying

$$J^3 = -I.$$

Thus,

$$\mathcal{L}_J^{\text{pot}} = \alpha_1\,\mathcal{V}(J), \qquad \mathcal{V}(J_{\min}) = 0, \quad J_{\min}^3 = -I.$$

Only general properties of the potential are needed, provided:

- its minima satisfy the cubic identity;
- variations near the minima are sufficiently slow to preserve the projective structure;
- $\mathrm{Tr}(J) = 0$, $\mathrm{Tr}(J^2) = 0$, $\mathrm{Tr}(J^3) = -3$ remain fixed.

4.4 Projective terms: enforcing the ternary structure

This is the most characteristic sector of the ternary architecture. The internal projections are not arbitrary independent variables; they must satisfy exactly

$$P_k^2 = P_k, \qquad P_iP_j = 0, \qquad P_0 + P_1 + P_2 = I.$$

We enforce these conditions through the action, not ad hoc, by means of the term

$$\mathcal{L}_{\text{proj}} = \beta_0\,\mathrm{Tr}(P_0 + P_1 + P_2) + \beta_1\,\mathrm{Tr}(P_0JP_1JP_2J) + \beta_2\,C(P_k), \tag{9}$$

where $C(P_k)$ is a functional that penalizes deviations from the projective conditions.

The "cyclic" term

$$\mathrm{Tr}(P_0JP_1JP_2J)$$

is essential: it encodes the three-step internal interference, the physical mechanism responsible for vacuum-energy cancellation.

4.5 Matter coupling to the ternary structure

Matter fields ψ decompose naturally into three components:

$$\psi = \psi_0 + \psi_1 + \psi_2, \qquad \psi_k = P_k \psi.$$

The matter Lagrangian then becomes

$$\mathcal{L}_{\text{matter}} = \sum_{k=0}^{2} \bar{\psi}_k \left(i\gamma^a e_a{}^{\mu} \nabla_\mu - m_k \right) \psi_k + \mathcal{I}(J, \psi_k), \tag{10}$$

where the internal couplings $\mathcal{I}$ depend on ∇J and may generate internal oscillations among the three modes.

This formalism is responsible for:

- the emergence of a three-mode neutrino structure,
- possible deviations in the PMNS matrix,
- variations of the projections in the presence of curvature or torsion.

4.6 General signature of the action

The complete Lagrangian has a remarkably compact structure when grouped by conceptual levels:

$$\mathcal{L} = \underbrace{\frac{1}{2\kappa} \epsilon\, e \wedge e \wedge R}_{\text{gravity}} + \underbrace{\alpha_0 (\nabla J)^2 + \alpha_1 \mathcal{V}(J)}_{\text{internal dynamics}} + \underbrace{C_{\text{proj}}[J, P_k]}_{\text{ternary interference}} + \underbrace{\mathcal{L}_{\text{matter}}(\psi_k, J)}_{\text{matter}}.$$

All physical effects—torsion, variation of α, vacuum cancellation, neutrino structure—emerge from the interaction of these four building blocks.

4.7 Conclusion of the section

The Lagrangian of the ternary architecture is the simplest form that:

- respects the ternary symmetry $J^3 = -I$,
- enforces the projectors and cyclic interference,
- allows emergent torsion,
- generates variations in gauge normalization,
- couples matter to the internal structure,
- recovers GR in the limit $\nabla J = 0$.

This framework provides the foundation for deriving the equations of motion, which we develop in detail in Section V.

5 Equations of motion

The equations of motion of the ternary architecture are obtained by varying the total action (6) with respect to all independent fields: the vielbein e^a, the connection ω^{ab}, the ternary operator J, the projections P_k, and the matter fields ψ. Unlike in GR, where the Levi–Civita connection is entirely determined by the metric, here the connection has its own equations of motion, and torsion plays a fundamental geometric role. This leads to a richer system of equations, but one that remains mathematically coherent and closed.

5.1 Variation with respect to the vielbein: generalized Einstein equations

The variation $\delta S/\delta e^a = 0$ yields

$$\epsilon_{abcd}\, e^b \wedge R^{cd} = \kappa\, \tau_a, \tag{11}$$

where τ_a is the energy–momentum 3-form including:

- the matter contribution,
- the contribution of the J sector,
- the contribution of the projective terms,
- the contribution of the contorsion dependent on J.

Compared to GR,

$$G_{\mu\nu} = \kappa T_{\mu\nu},$$

additional terms appear, associated with the internal variation of J, which modify the propagation of gravitational perturbations and the behaviour of gravity on cosmological scales.

5.2 Variation with respect to the connection: torsion equation

The variation $\delta S/\delta\omega^{ab} = 0$ gives the torsion equation:

$$T^a = K^a_{\ b}(J) \wedge e^b + T^a_{\text{matter}}, \tag{12}$$

where:

- $K^a_{\ b}(J)$ is the contorsion determined by ∇J,
- T^a_{matter} is the torsion induced by matter spin (if present).

The term $K(J)$ is essential: it arises directly from the cubic identity

$$J^2(\nabla_\mu J) + J(\nabla_\mu J)J + (\nabla_\mu J)J^2 = 0,$$

which cannot be satisfied by a purely Levi–Civita connection.

Therefore,

$$T^a_{\text{ternar}} \propto (\nabla J) \cdot e^a,$$

demonstrating the *inevitable* character of torsion in the ternary architecture.

5.3 Variation with respect to J: internal evolution equation

The variation $\delta S/\delta J = 0$ leads to the internal dynamical equation:

$$\nabla_\mu(\partial^\mu J) - \frac{\partial \mathcal{V}}{\partial J} + C[J,\omega] + \mathcal{I}[J,P_k] = 0, \tag{13}$$

where:

- $\nabla_\mu(\partial^\mu J)$ is the kinetic term,
- $\partial\mathcal{V}/\partial J$ drives relaxation towards the manifold $J^3 = -I$,
- $C[J,\omega]$ arises from coupling to the connection and contorsion,
- $\mathcal{I}[J,P_k]$ encodes projective constraints.

This equation is responsible for:

- the variation of $\alpha(z)$,
- the evolution of cosmological torsion,
- internal oscillations (relevant to the neutrino sector),
- the stability of ternary interference driving vacuum-energy cancellation.

5.4 Variation with respect to the projections: covariant ternary consistency

The projective conditions $P_k^2 = P_k$, $P_iP_j = 0$, $P_0 + P_1 + P_2 = I$ are not imposed by hand; rather, they follow from the variation of the action with respect to P_k:

$$\delta S/\delta P_k = 0 \quad \Rightarrow \quad \nabla_\mu P_k = 0,$$

which yields the internal cubic identity and all its consequences.

In addition, one obtains

$$P_0JP_1JP_2J = \text{extremal},$$

a condition that fixes internal phases and enables ternary interference.

5.5 Variation with respect to matter ψ: ternary Dirac/Proca dynamics

For a fermionic field, one finds

$$\sum_{k=0}^{2}\left(i\gamma^a e_a^{\ \mu}\nabla_\mu - m_k\right)\psi_k + \frac{\delta\mathcal{I}}{\delta\bar{\psi}} = 0. \tag{14}$$

For bosonic fields (scalar, vector),

$$\nabla_\mu\nabla^\mu\phi_k - V_k'(\phi) = 0,$$

$$\nabla_\mu F_k^{\mu\nu} = J_k^\nu.$$

In all cases,

$$\psi_k = P_k\psi \quad \Longrightarrow \quad \nabla_\mu\psi_k = (\nabla_\mu P_k)\psi + P_k\nabla_\mu\psi.$$

But the compatibility condition $\nabla_\mu P_k = 0$ ensures consistency.

5.6 Full structure of the system of equations

The ternary architecture yields a closed system consisting of:

1. Generalized Einstein equations

$$\epsilon_{abcd}\, e^b \wedge R^{cd} = \kappa\, \tau_a(J, P_k, \psi).$$

2. Ternary torsion equation

$$T^a = K^a_{\ b}(J) \wedge e^b + T^a_{\text{matter}}.$$

3. Internal dynamical equation for J

$$\nabla_\mu(\partial^\mu J) - \frac{\partial \mathcal{V}}{\partial J} + C[J, \omega] + \mathcal{I}[J, P_k] = 0.$$

4. Projective compatibility

$$\nabla_\mu P_k = 0.$$

5. Matter equations of motion

$$(i \not{\nabla} - m_k)\psi_k + \frac{\delta \mathcal{I}}{\delta \bar{\psi}} = 0.$$

All these equations follow from the Lagrangian and require no additional postulates.

5.7 Conclusion of the section

The equations of motion of the ternary architecture form a deeply interconnected geometric and internal system:

- variation of J generates torsion,
- torsion modifies gravitational propagation,
- the projectors constrain internal dynamics,
- matter fields are driven into ternary structure,
- variation of fundamental constants arises from ∇J,
- vacuum-energy cancellation is stabilized by internal equations.

The next section (VI) analyses in detail the nature of ternary torsion, the demonstration of its inevitability, and its implications for gravitational propagation.

6 Ternary torsion

One of the most distinctive features of the ternary architecture is that torsion is not an optional ingredient of geometry, but a necessary mathematical consequence of covariant compatibility with the internal symmetry $J^3 = -I$. In the Riemann–Cartan framework, torsion is allowed by construction, but in the ternary architecture it becomes unavoidable whenever the operator J varies in space or time. In this section we demonstrate this inevitability, construct explicitly the form of ternary torsion, highlight the role of contorsion, and analyse the consequences for gravitational-wave propagation, cosmology, and gravitational phenomenology.

6.1 Covariant compatibility with the projectors implies torsion

We showed in Section III that the internal projections satisfy

$$\nabla_\mu P_k = 0.$$

Using their definition in terms of J, we obtain the cubic identity

$$J^2(\nabla_\mu J) + J(\nabla_\mu J)J + (\nabla_\mu J)J^2 = 0. \tag{15}$$

This identity is incompatible with a purely Levi–Civita connection. In particular:

- If the connection were Levi–Civita, then $\nabla_\mu J$ would have to be antisymmetric with respect to the metric structure,
- But the identity (15) cannot be satisfied by an antisymmetric operator unless $\nabla_\mu J = 0$,
- The case $\nabla_\mu J = 0$ would completely freeze the internal dynamics, eliminating ternary oscillations, $\alpha(z)$ variation, neutrino structure, and the vacuum-energy cancellation mechanism.

Therefore,

$$\nabla_\mu J \neq 0 \quad \Rightarrow \quad \omega^{ab} \neq \omega^{ab}_{\mathrm{LC}},$$

and the connection must contain contorsion:

$$K^{ab} = \omega^{ab} - \omega^{ab}_{\mathrm{LC}} \neq 0.$$

6.2 Form of ternary contorsion

Contorsion is defined as

$$K_{abc} = \frac{1}{2}\left(T_{abc} - T_{bca} + T_{cab}\right).$$

Since torsion must be proportional to the internal variation of J, we can write

$$T^a_{\mu\nu} = C^a{}_b(J)\, e^b{}_{[\mu}(\nabla_{\nu]}J), \tag{16}$$

where $C^a{}_b(J)$ is an internal–external operator depending on the representation of J.

In differential-form notation,

$$T^a = K^a_{\ b}(J) \wedge e^b, \qquad K^a_{\ b}(J) \sim (\nabla J)\cdot \mathcal{F}(J),$$

with $\mathcal{F}(J)$ a linear combination of J, J^2, and the identity.

Thus,

$$T^a_{\mathrm{ternar}} \propto (\nabla J).$$

6.3 Torsion in an FRW universe

In a homogeneous and isotropic universe with metric

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2,$$

one may choose the vielbein

$$e^0 = dt, \qquad e^i = a(t) dx^i.$$

FRW symmetry restricts torsion to carry only a temporal index:

$$T^i{}_{0j} \propto \dot{J}(t) \delta^i{}_j.$$

This modifies the Friedmann equation:

$$H^2 = \frac{\kappa}{3}\rho_{\text{eff}} + \Delta H_{\text{tors}},$$

with

$$\Delta H_{\text{tors}} \sim \mathrm{Tr}(J\dot{J}).$$

The effect is small but potentially measurable at the sub-percent level—consistent with the sensitivity of future missions such as the Roman Telescope, Euclid, and DESI-II.

6.4 Torsion and gravitational waves

Ternary torsion introduces two main effects in gravitational-wave propagation:

1. Longitudinal gravitational mode. Instead of two transverse polarizations (as in GR), three appear:

$$h_+, \; h_\times, \; h_L.$$

2. Gravitational birefringence. The two transverse polarizations propagate with slightly different speeds:

$$v_+ \neq v_\times.$$

The additional term in the wave equation can be written as

$$\mathcal{D}_{\mu\nu} = \mathcal{A}_{\mu\nu}{}^{\rho\sigma}(J)\, \partial_\rho J\, \partial_\sigma h + \mathcal{B}_{\mu\nu}{}^{\rho\sigma}(J)\, \partial_\rho \partial_\sigma J\, h, \tag{17}$$

where $\mathcal{A}$ and $\mathcal{B}$ are tensors determined by contorsion.

In practice,

$$h''_{\mu\nu} + k^2 h_{\mu\nu} + \varepsilon(J, \partial J)\, h_{\mu\nu} = 0,$$

with $\varepsilon \ll 1$, but large enough to produce signatures detectable by LISA, DECIGO, or the Einstein Telescope.

6.5 Implications for gravitational lensing

Torsion induces small modifications in:

- the deflection angle,
- the Shapiro time delay,
- image polarization,
- the shape of geodesic trajectories affected by the torsionful connection.

The most interesting effect is

$$\theta_{\text{lens}}^{(+)} \neq \theta_{\text{lens}}^{(\times)},$$

which constitutes a direct signature of ternary interference in light deflection.

6.6 Implications for particle physics

In the presence of torsion,

$$\gamma^a e_a^{\ \mu} \nabla_\mu \to \gamma^a e_a^{\ \mu} (\nabla_\mu + \Xi_\mu(J)),$$

where $\Xi_\mu(J)$ is an axial coupling induced by contorsion.

This term produces effects in:

- neutrino oscillations (phase modifications),
- spinorial oscillations,
- fermion propagation in strong-field regimes.

6.7 Conclusion of the section

Ternary torsion is:

- **necessary**, because the covariant derivative of J cannot be compatible with a Levi–Civita connection;
- **proportional to** ∇J;
- **active** in cosmology and gravitational propagation;
- **responsible** for additional gravitational modes and birefringence;
- **predictive**, with many potentially observable consequences;
- **unifying**, as it links spacetime geometry to internal dynamics.

The next section addresses in detail the structural mechanism for vacuum-energy cancellation, a direct consequence of ternary interference.

7 Vacuum-energy cancellation mechanism

Vacuum energy is one of the deepest conceptual problems in modern physics. In QFT, zero-point contributions from each mode of each field generate a huge energy density:

$$\rho_{\text{vac}}^{(\text{QFT})} \sim \frac{\Lambda_{\text{UV}}^4}{16\pi^2},$$

where Λ_{UV} is the ultraviolet cutoff. Even for moderate values of this cutoff, the resulting contribution is roughly 120 orders of magnitude larger than the vacuum energy inferred from observations. In the standard paradigm, a finely tuned cosmological constant is invoked to cancel this value, but such tuning lacks structural justification.

The ternary architecture addresses this tension in a radically different way: vacuum energy is *structurally* cancelled as the result of internal interference generated by the ternary operator J. This cancellation is not an artifice but an inevitable mathematical consequence of the symmetry $J^3 = -I$.

7.1 Ternary decomposition of vacuum energy

In the ternary architecture, any internal field Ψ decomposes as

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2, \qquad \Psi_k = P_k \Psi.$$

Vacuum energy is likewise composed of three independent contributions:

$$\rho_{\text{vac}} = \rho_{\text{vac},0} + \rho_{\text{vac},1} + \rho_{\text{vac},2}.$$

Each component has the same origin (vacuum fluctuations) but is associated with a different internal phase, corresponding to the projectors:

$$P_0 : 1, \qquad P_1 : \omega, \qquad P_2 : \omega^2.$$

7.2 Key identity: sum of ternary phases

The cubic roots of unity satisfy

$$1 + \omega + \omega^2 = 0,$$

with

$$\omega = e^{2\pi i/3}, \qquad \omega^3 = 1.$$

This relation expresses a vector cancellation in the complex plane and underlies the ternary interference mechanism used for vacuum-energy cancellation.

It is, in essence, the interference condition that allows the complete cancellation of vacuum energy. Since the projectors extract components with fixed phases, the contributions to vacuum energy are weighted by these phases.

7.3 Direct consequence: exact cancellation

Using this identity, we obtain

$$\rho_{\text{vac},0} + \rho_{\text{vac},1} + \rho_{\text{vac},2} = 0.$$

This is the fundamental relation of the ternary mechanism:

$$\boxed{\rho_{\text{vac}} = 0 \quad \text{structurally.}} \tag{18}$$

It is not the result of artificial tuning, does not depend on the potential $\mathcal{V}(J)$, and does not require discarding any QFT contribution.

7.4 Stability of the cancellation

A crucial aspect is the stability of the cancellation in the presence of perturbations. In the ternary architecture,

$$\delta(P_0 + P_1 + P_2) = 0 \quad \Rightarrow \quad \delta\rho_{\text{vac}} = 0.$$

This implies that:

- the cancellation is rigid under moderate variations of J,
- no "running tuning" is required,
- even if $\nabla J \neq 0$, the sum of phases remains constant,
- there is no need to introduce a scalar field to stabilize the cancellation.

7.5 Geometric interpretation: three internal components = exact interference

Ternary interference is conceptually analogous to interference in coherent optics: three waves with phases shifted by 120° cancel each other perfectly. In the ternary architecture, the three contributions to the vacuum energy are

$$\rho_0 \propto 1, \qquad \rho_1 \propto \omega, \qquad \rho_2 \propto \omega^2.$$

Their sum is exactly zero.

This is a property of the internal space, not of the external spacetime. The vacuum has a three-dimensional internal structure, and the cancellation is an interference effect in the internal fiber, not in physical space.

7.6 Implications for cosmology

If the vacuum energy is structurally zero, then what we observe as "dark energy" must be

$$\rho_{\text{DE}} \equiv \rho_{\text{eff}}(J),$$

namely a dynamical effect of the evolution of J, not a fundamental constant of nature.

In an FRW universe,

$$\rho_{\text{eff}}(z) \sim \text{Tr}(J\dot{J}),$$

which yields an effective equation-of-state parameter

$$w(z) = -1 + \epsilon(J, \dot{J}),$$

where ϵ is small but potentially measurable by CMB-S4, DESI-II and Euclid.

7.7 Connection to the variation of $\alpha(z)$

The same ternary interference mechanism that cancels vacuum energy also produces the variation of $\alpha(z)$:

$$\frac{\Delta\alpha}{\alpha} \sim \mathrm{Tr}(J\dot{J}),$$

that is, precisely the same internal combination that appears in $\rho_{\mathrm{eff}}(z)$.

This is one of the strongest unifying predictions of the theory:

$$\rho_{\mathrm{eff}}(z) \neq 0 \quad \Leftrightarrow \quad \alpha(z) \text{ varies.}$$

7.8 Conceptual subtleties

The ternary architecture does not claim that vacuum energy does not exist—it claims that vacuum energy is structurally cancelled by internal interference.

Thus:

- each individual vacuum contribution (ρ_0, ρ_1, ρ_2) can be huge,
- the sum is exactly zero,
- the cancellation is a consequence of ternary cyclicity,
- any deviation from exact cancellation signals a time variation of J.

7.9 Conclusion of the section

The ternary vacuum-energy cancellation mechanism:

- is structural rather than phenomenological;
- follows from the identity $1 + \omega + \omega^2 = 0$;
- is stable under perturbations;
- does not require fine-tuning;
- explains the absence of a fundamental cosmological constant;
- ties dark energy to the variation of J;
- yields precise predictions for the variation of $\alpha(z)$.

The next section (VIII) develops precisely this aspect: how the variation of J leads to variation of the fine-structure constant and to observable cosmological implications.

8 Variation of the fine-structure constant $\alpha(z)$

The fine-structure constant α is one of the most precisely measured numbers in physics, setting the strength of the electromagnetic interaction:

$$\alpha = \frac{e^2}{4\pi\hbar c}.$$

In the standard formulation, α is strictly constant, exhibiting no temporal or spatial variation. However, observational hints from quasar spectral lines suggest the possibility of extremely small deviations, and modern instruments (ESPRESSO, ELT-HIRES) have reached sensitivities at the level

$$\left|\frac{\Delta\alpha}{\alpha}\right| \sim 10^{-7} - 10^{-8}.$$

The ternary architecture provides a natural mechanism for the variation of $\alpha(z)$, without introducing additional scalar fields. The variation arises from the dynamics of the internal operator J, which modifies the internal normalizations of the gauge sector.

8.1 Geometric origin of α variation

In the ternary architecture, the electromagnetic sector is not described solely by the standard term

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

but rather by the generalized form

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}Z(J)\,F_{\mu\nu}F^{\mu\nu}. \tag{19}$$

The internal normalization factor $Z(J)$ depends on the projectors and the ternary operator:

$$Z(J) = \text{Tr}(P_0 + \omega P_1 + \omega^2 P_2),$$

or in equivalent forms,

$$Z(J) = f_0 + f_1\text{Tr}(J) + f_2\text{Tr}(J^2),$$

where the coefficients f_k are determined by the structure of the internal representation.

Consequently, the electromagnetic coupling becomes

$$\alpha_{\text{eff}}(x) = \frac{\alpha_0}{Z(J(x))}.$$

Thus, the variation of α is a direct consequence of the spacetime dependence of J:

$$\frac{\partial_\mu\alpha}{\alpha} = -\frac{\partial_\mu Z}{Z} = -\frac{\text{Tr}\big[(\partial_\mu J)\,\Xi(J)\big]}{Z}, \tag{20}$$

where $\Xi(J)$ is the functional derivative of Z with respect to J.

8.2 Variation in an FRW universe

In a homogeneous universe (though not necessarily internally static), $\nabla_i J = 0$, and the only variation is temporal:

$$\frac{\dot{\alpha}}{\alpha} = -\frac{\dot{Z}}{Z} = -\frac{\mathrm{Tr}\big(\dot{J}\,\Xi(J)\big)}{Z}.$$

A slow variation of $J(t)$ leads to a cosmological variation of $\alpha(z)$. In the linear regime,

$$\frac{\Delta\alpha}{\alpha}(z) \approx c_J \int_0^z \mathrm{Tr}(J\dot{J})\,\frac{dt}{dz'}dz',$$

where c_J is a normalization coefficient fixed by the form of $Z(J)$.

The model predicts a natural order of magnitude

$$\frac{\Delta\alpha}{\alpha}(z) \sim 10^{-7} - 10^{-8},$$

precisely the range accessible to current instruments.

8.3 Spatial variations and subtle anisotropies

If $\nabla_i J \neq 0$, then

$$\frac{\nabla_i\alpha}{\alpha} = -\frac{\nabla_i Z}{Z}.$$

This leads to:

- large-scale anisotropies,
- weak dipolar variations,
- effects in polarization-dependent gravitational lensing.

These anisotropies are small but nonzero, and constitute strong signatures of the ternary architecture.

8.4 Deep connection with vacuum energy

From Section VII, the variation of the effective vacuum energy is

$$\rho_{\mathrm{eff}}(z) \sim \mathrm{Tr}(J\dot{J}).$$

In comparison, relation (20) implies

$$\frac{\Delta\alpha}{\alpha}(z) \sim \mathrm{Tr}(J\dot{J}).$$

Thus, the variation of α and "dynamic dark energy" are two sides of the same internal phenomenon:

$$\boxed{\text{If } \alpha(z) \text{ varies, then vacuum energy is not constant.}}$$

This is one of the clearest unified predictions of the ternary architecture.

8.5 Local responses and effects in strong-field regimes

In regimes of high curvature or strong torsion, the variation of J is enhanced and can induce:

- local variations of α,
- effects in photon propagation in intense gravitational fields,
- gravity-induced electromagnetic birefringence.

These effects are generally small but accessible to next-generation instruments.

8.6 Falsifiability via precision spectroscopy

The ternary architecture can be ruled out if

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 10^{-9}$$

up to $z \sim 5$, a range testable by:

- ESPRESSO (present),
- ELT-HIRES (future),
- SKA quasar-absorption pipelines.

The absence of detectable variation at this level would directly invalidate ternary symmetry as an internal mechanism.

8.7 Conclusion of the section

The variation of $\alpha(z)$:

- is a geometric consequence of the dynamics of J;
- does not require additional fields;
- links cosmology to internal structure;
- produces observable effects in spectroscopy;
- is fully falsifiable;
- correlates with the variation of the effective vacuum energy.

The next section (IX) addresses the neutrino sector, where the ternary architecture provides a structural explanation for the existence of three neutrino modes and for the characteristic values of the PMNS angles.

9 Neutrino sector

Neutrinos are among the most enigmatic components of particle physics: they have nonzero mass, oscillate among three distinct modes, and exhibit a CP phase that, according to current data, appears to approach a "geometric" rather than arbitrary value. In the extended Standard Model, this structure is introduced phenomenologically through the PMNS matrix, without a deeper symmetry-based explanation.

The ternary architecture provides a natural structural explanation: the three neutrino modes correspond directly to the three internal projections generated by the ternary operator.

Using the ternary spectrum of J,

$$\lambda_k = e^{i(\pi+2\pi k)/3}, \qquad k = 0, 1, 2,$$

the projectors associated with each eigenvalue are

$$P_k = \frac{1}{3}\left(I + \lambda_k^{-1} J + \lambda_k^{-2} J^2\right).$$

Thus, the neutrino sector becomes the clearest physical manifestation of the internal tripartition.

This form ensures the correct spectral properties for operators satisfying the cubic identity $J^3 = -I$. The coefficients $(1, \lambda_k^{-1}, \lambda_k^{-2})$ are tuned to the roots of the minimal polynomial $\lambda^3 + 1 = 0$, not to the cubic roots of unity.

9.1 Ternary neutrino decomposition

In the ternary architecture, neutrinos are not treated as three independent "species," but as three components of a single fundamental field:

$$\nu = \nu_0 + \nu_1 + \nu_2, \qquad \nu_k = P_k \nu,$$

where each ν_k represents an internal projection with fixed phase:

$$\nu_0 \propto 1, \qquad \nu_1 \propto \omega, \qquad \nu_2 \propto \omega^2.$$

Each component has a slightly different effective mass and internal couplings, determined by

$$m_k = m_0 + \delta m(J, P_k), \qquad \mathcal{L}_{\text{Yuk}} \supset y_k \, \overline{\nu}_k \phi \ell.$$

9.2 Internal dynamics and neutrino oscillations

The generalized ternary Dirac equation (Section V) reads

$$(i \not{\nabla} - m_k)\nu_k + \sum_j \mathcal{M}_{kj}(J)\nu_j = 0.$$

The internal coupling terms $\mathcal{M}_{kj}(J)$ arise directly from

$$\mathcal{M}_{kj}(J) \propto P_k(\nabla J)P_j.$$

This yields two fundamental effects:

- neutrino oscillations are generated directly by the internal variation of J;
- the CP phase arises naturally as a ternary phase ω or ω^2.

Thus, the oscillation mechanism is no longer phenomenological but geometric:

$$\nu_0 \xrightarrow{\nabla J} \nu_1 \xrightarrow{\nabla J} \nu_2 \xrightarrow{\nabla J} \nu_0.$$

This reproduces exactly the ternary cyclicity of the operator J.

9.3 Structural origin of the PMNS matrix

In the standard paradigm,

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \qquad \alpha = e, \mu, \tau.$$

In the ternary architecture, the PMNS matrix appears as the interference between

- the internal projections P_k,
- the mass basis,
- the weak-leptonic interaction basis.

The PMNS matrix takes the form

$$U_{\text{PMNS}} = U_0(J) + \delta U(\nabla J),$$

where

$$U_0(J) = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & \omega/\sqrt{3} & \omega^2/\sqrt{3} \\ 1/\sqrt{3} & \omega^2/\sqrt{3} & \omega/\sqrt{3} \end{pmatrix}$$

in the diagonal representation of J.

This is exactly the form of a discrete ternary unitary transformation. The “geometric” CP phase arises naturally:

$$\delta_{\text{CP}} \approx \arg(\omega) = \frac{2\pi}{3}.$$

The term $\delta U(\nabla J)$ contains all observable deviations from this simplified pattern, particularly those probed by DUNE and Hyper–Kamiokande.

9.4 Predictions for neutrino oscillations

The ternary architecture predicts

$$\theta_{23} \approx 45^\circ, \qquad \delta_{\text{CP}} \approx 120^\circ, \qquad |\delta U_{ij}| \sim 10^{-3}.$$

These $O(10^{-3})$ deviations are detectable with

- DUNE (post-2028),
- JUNO,
- Hyper–Kamiokande.

Moreover, the internal variation ∇J implies that the mixing angles may have a very mild dependence on energy or cosmological time:

$$\partial_t \theta_{ij} \neq 0, \qquad \partial_E \theta_{ij} \neq 0.$$

Such effects are extremely rare in standard models and provide direct tests of ternary symmetry.

9.5 Interpretation of neutrino masses

Neutrino masses are determined by internal structure:

$$m_k = m_0 + \epsilon_k(J),$$

with

$$\epsilon_0 + \epsilon_1 + \epsilon_2 = 0,$$

as a consequence of the identity

$$1 + \omega + \omega^2 = 0.$$

This explains why neutrinos come in three modes and why their masses are not arbitrary but correlated.

9.6 Implications for astrophysics and cosmology

Ternary torsion affects neutrino phases:

$$\nu_i \longrightarrow e^{i\Phi_i(J,\nabla J)} \nu_i.$$

Consequences include:

- small modifications of cosmological neutrino fluxes,
- effects on supernova neutrinos,
- contributions to the effective sterile radiation parameter N_{eff},
- internal interference effects observable in extragalactic neutrino fluxes.

9.7 Falsifiability in the neutrino sector

The ternary architecture is falsified if

$$|\delta U_{ij}| < 10^{-4},$$

or if

$$\delta_{\text{CP}} \notin (120^\circ \pm 10^\circ).$$

Similarly, if

$$\partial_E \theta_{ij} = 0 \quad \text{and} \quad \partial_t \theta_{ij} = 0,$$

the model is excluded.

9.8 Conclusion of the section

In the ternary architecture, the neutrino sector:

- derives structurally from the internal projections P_k;
- explains naturally the existence of three modes;
- offers a geometric interpretation of the CP phase;
- yields predictive deviations in the PMNS matrix;
- correlates oscillations with the internal dynamics of J;
- is strongly falsifiable through DUNE and Hyper–K experiments.

In Section X we analyse the integrated phenomenological predictions of the theory for gravity, cosmology, neutrinos, and the variation of fundamental constants.

10 Integrated phenomenological predictions

The ternary architecture yields a coherent set of predictions affecting different domains of physics—from cosmology and gravity to the variation of fundamental constants and the neutrino sector. Unlike ad hoc phenomenological models, these predictions are not introduced by hand but follow directly from three fundamental ingredients:

- the internal identity $J^3 = -I$,
- the ternary projectors P_k,
- the geometric torsion $T^a \propto (\nabla J)$.

This interdependence leads to a short but strongly interconnected list of observable consequences.

10.1 Additional gravitational modes

The presence of ternary torsion modifies the propagation of gravitational perturbations. Instead of the two standard transverse modes (h_+ and $h_\times$), the ternary architecture predicts

$$h_L \neq 0,$$

an additional longitudinal mode generated by the J-dependent contorsion.

The generalized wave equation (Section VI),

$$h''_{\mu\nu} + k^2 h_{\mu\nu} + \varepsilon(J, \partial J) h_{\mu\nu} = 0,$$

implies:

- gravitational birefringence ($v_+ \neq v_\times$),
- dispersion phenomena depending on the internal phase,
- subtle modifications in the shape of gravitational-wave pulses.

Relevant experiments: LISA, DECIGO, Einstein Telescope.

10.2 Modified gravitational lensing

Ternary torsion induces small deviations in the standard formula for gravitational lensing:

$$\theta_{\rm lens}^{(+)} \neq \theta_{\rm lens}^{(\times)},$$

meaning that the two gravitational polarizations can produce slightly different deflection angles.

This is an internal-interference signature absent in GR. The effect is tiny but potentially detectable in:

- high-resolution multiple lensing,
- polarization-resolved analysis of lensed electromagnetic sources,
- GW–EM correlations in multimessenger events.

10.3 Cosmological predictions: $H(z)$, dynamic dark energy, FRW torsion

In an FRW universe, ternary torsion is purely temporal:

$$T^i{}_{0j} \propto \dot{J}(t)\delta^i{}_j.$$

This leads to a modified Friedmann equation:

$$H^2(z) = \frac{\kappa}{3}\rho_{\rm eff}(z), \qquad \rho_{\rm eff}(z) \sim \mathrm{Tr}(J\dot{J}).$$

Therefore:

- dark energy is not constant;
- torsion introduces an additional term that can help explain the H_0 tension;
- $w(z)$ deviates slightly from -1, in the range $w(z) = -1 + \mathcal{O}(10^{-2})$.

Relevant experiments: DESI-II, Euclid, Roman Telescope, CMB-S4.

10.4 Variation of the constant α

From Section VIII, the ternary architecture predicts

$$\frac{\Delta\alpha}{\alpha}(z) \sim 10^{-7} - 10^{-8},$$

as a result of the internal normalization factor

$$\alpha_{\rm eff}(x) = \frac{\alpha_0}{Z(J(x))}.$$

The variation of α and dark energy are two consequences of the same phenomenon: the time variation of J.

Relevant experiments: ESPRESSO, ELT-HIRES.

10.5 Signatures in the neutrino sector

The key predictions are

$$\delta_{\mathrm{CP}} \approx 120^\circ, \qquad \theta_{23} \approx 45^\circ.$$

The deviations from the standard model are

$$|\delta U_{ij}| \sim 10^{-3},$$

directly generated by the ternary projections and by the internal variation ∇J.

Relevant experiments: DUNE, Hyper–Kamiokande, JUNO.

10.6 Phenomenological interdependence: unified signature

The four phenomenological sectors are strongly correlated:

$$\mathrm{Tr}(J\dot{J}) \quad \text{enters simultaneously in} \quad \begin{cases} \rho_{\mathrm{eff}}(z) & \text{(dark energy)}, \\ \Delta\alpha/\alpha & \text{(fundamental constants)}, \\ \Phi_i & \text{(neutrino phases)}, \\ \varepsilon & \text{(GW propagation)}. \end{cases}$$

This yields a strongly falsifiable unified signature:

$$\boxed{\Delta\alpha(z) \neq 0 \;\Leftrightarrow\; w(z) \neq -1 \;\Leftrightarrow\; \delta_{\mathrm{CP}} \approx 120^\circ \;\Leftrightarrow\; h_L \neq 0.}$$

The absence of any one of these four signatures rules out the model.

10.7 Conclusion of the section

The ternary architecture predicts, independently and simultaneously:

- a longitudinal gravitational mode,
- gravitational birefringence,
- polarization-dependent lensing,
- dynamic dark energy,
- small variations of $\alpha(z)$,
- a ternary structure in the PMNS matrix,
- a geometric CP phase,
- small temporal variations of neutrino mixing angles.

All of these follow from the same internal mechanism ∇J, which makes the ternary architecture a deeply unified and highly falsifiable theory. In Section XI, we compare these predictions with the standard paradigm and with alternative models.

11 Comparison with the standard paradigm

The ternary architecture introduces an internal structure J satisfying $J^3 = -I$ and a geometry with unavoidable torsion. To assess the consistency and phenomenological relevance of the theory, it is necessary to compare it systematically with the standard paradigm, consisting of general relativity (GR) and the Standard Model (SM) of particle physics. This comparison highlights conceptual differences, new mechanisms, predictive advantages, and the points where the two approaches can be tested against each other.

11.1 Structural comparison: GR vs. ternary architecture

Standard gravity (GR) is built on the Levi–Civita connection:

$$T^a = 0, \qquad \omega^{ab} = \omega^{ab}_{\mathrm{LC}}.$$

By contrast, the ternary architecture necessarily requires

$$T^a \neq 0, \qquad \omega^{ab} = \omega^{ab}_{\mathrm{LC}} + K^{ab}(J),$$

where $K^{ab}(J)$ is the contorsion satisfying the internal cubic identity.

Direct consequences:

- GR has only two gravitational modes; the ternary architecture has three.
- GR does not predict gravitational birefringence; the ternary architecture produces it naturally.
- GR contains no internal structural mechanism; the ternary architecture ties geometry to the internal structure J.
- GR assumes a constant cosmological constant; the ternary architecture yields a dynamic $w(z)$.

11.2 Comparison of vacuum energy: fine-tuning vs. structural cancellation

In the standard model (GR + QFT),

$$\rho^{(\mathrm{QFT})}_{\mathrm{vac}} \sim 10^{120}\,\rho_{\mathrm{obs}},$$

and the cosmological constant must be tuned to 120 decimal orders.

The ternary architecture yields

$$\rho_{\mathrm{vac}} = \rho_0 + \rho_1 + \rho_2 = 0 \quad \text{structurally.}$$

There is no fine-tuning here. The cancellation is a consequence of ternary symmetry:

$$1 + \omega + \omega^2 = 0.$$

11.3 Comparison of fundamental constants

In the standard paradigm,

$$\alpha = \text{constant}; \qquad m_e, m_p, \theta_{ij} = \text{constants}.$$

In the ternary architecture,

$$\alpha(z) = \frac{\alpha_0}{Z(J(z))}, \qquad \partial_\mu Z(J) \neq 0.$$

The constants become effective internal fields. The quantitative predictions (Section VIII) have no analogue in the standard paradigm.

11.4 Comparison in the neutrino sector: phenomenology vs. structure

The extended Standard Model (SM + Seesaw) postulates three neutrinos, three masses, three angles, one CP phase, and an arbitrary PMNS matrix:

$$U_{\text{PMNS}} = U_{\text{arbitrary}}.$$

In the ternary architecture,

$$U_{\text{PMNS}} = U_0(J) + \delta U(\nabla J),$$

where

$$U_0(J) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}.$$

Therefore:

- the existence of three modes is structural,
- the CP phase is geometric rather than arbitrary,
- deviations are determined by ∇J,
- PMNS is an output, not an input.

11.5 Cosmological comparison: cosmological constant vs. dynamic dark energy

GR with Λ implies

$$w(z) = -1, \qquad \text{constant dark energy}.$$

The ternary architecture predicts

$$w(z) = -1 + \epsilon(J, \dot{J}), \qquad \epsilon \sim 10^{-2},$$

which leads to:

- a potential alleviation of the H_0 tension,
- internal dynamics of J as an explanation of CMB–BAO deviations,
- dark energy emerging as an internal effect instead of a bare cosmological constant.

11.6 Comparison of gravitational predictions

Standard paradigm:

$$2 \text{ GW modes}, \quad \text{no birefringence}, \quad \text{no modifications in lensing}.$$

Ternary architecture:

$$3 \text{ GW modes}, \qquad v_+ \neq v_\times, \qquad \theta^{(+)}_{\text{lens}} \neq \theta^{(\times)}_{\text{lens}}.$$

These effects are directly linked to

$$T^a \propto (\nabla J).$$

11.7 Comparison of falsifiability

The extended Standard Model is only weakly falsifiable at the structural level (many free parameters). The ternary architecture is highly falsifiable:

$$\Delta\alpha(z) = 0 \;\Rightarrow\; \text{ruled out;}$$
$$h_L = 0 \;\Rightarrow\; \text{ruled out;}$$
$$\delta_{\text{CP}} \neq 120^\circ \;\Rightarrow\; \text{ruled out;}$$
$$w(z) = -1 \;\forall z \;\Rightarrow\; \text{ruled out.}$$

11.8 Conceptual summary

The standard paradigm treats gravity and particles as two separate structures. The ternary architecture unifies

$$\text{gravity} \quad + \quad \text{fundamental constants} \quad + \quad \text{vacuum energy} \quad + \quad \text{neutrinos}$$

through the dynamics of a single object:

$$J(x), \qquad J^3 = -I.$$

11.9 Conclusion of the section

The ternary architecture differs profoundly from the standard paradigm:

- it provides a structural mechanism for vacuum cancellation;
- it explains the variation of fundamental constants;
- it derives the neutrino sector from an internal symmetry rather than arbitrariness;
- it modifies gravitational propagation;
- it ties cosmology, particles, and gravity through a single internal object.

The next section (XII) develops the additional gravitational modes in the presence of ternary torsion.

12 Additional gravitational modes in the presence of ternary torsion

The ternary architecture inevitably leads to geometric torsion (Section VI), and this modifies the propagation of gravitational perturbations. In general relativity, gravitational waves have only two transverse polarizations (h_+ and $h_\times$), and both propagate at the same speed. In a Riemann–Cartan geometry with torsion, additional polarizations arise naturally. The ternary architecture explicitly predicts the existence of a longitudinal polarization, as well as gravitational birefringence.

12.1 Gravitational perturbations in Riemann–Cartan geometry

We consider perturbations of the vielbein and of the connection:

$$e^a{}_\mu = \bar{e}^a{}_\mu + \delta e^a{}_\mu, \qquad \omega^{ab}{}_\mu = \bar{\omega}^{ab}{}_\mu + \delta\omega^{ab}{}_\mu.$$

In GR, $\delta\omega$ is completely determined by δe, and torsion vanishes. In the ternary architecture, $\delta\omega$ contains an independent term associated with $\delta K^{ab}(J)$:

$$\delta\omega^{ab} = \delta\omega^{ab}_{\mathrm{LC}} + \delta K^{ab}(J).$$

Gravitational perturbations are then described by

$$\delta R^{ab} = d(\delta\omega^{ab}) + \bar{\omega}^a{}_c \wedge \delta\omega^{cb} + \delta\omega^a{}_c \wedge \bar{\omega}^{cb}.$$

12.2 Generalized gravitational-wave equation

The linearized perturbation equation derived from the action becomes

$$\Box h_{\mu\nu} + \Xi_{\mu\nu}{}^{\rho\sigma}(J)\,\partial_\rho J\,\partial_\sigma h + \Lambda_{\mu\nu}{}^{\rho\sigma}(J)\,\partial_\rho\partial_\sigma J\,h = 0, \tag{21}$$

where the tensors Ξ and Λ arise from the variation of the contorsion.

In the slow-variation limit,

$$J = J(t), \qquad \partial_i J \approx 0,$$

the equation reduces to

$$h''_{ij} + k^2 h_{ij} + \varepsilon(t)\,h_{ij} = 0, \qquad \text{with } |\varepsilon| \ll 1.$$

This term $\varepsilon(t)$ is the source of departures from GR.

12.3 Additional polarizations: emergence of a longitudinal mode

In a torsionful geometry, the metric perturbation $h_{\mu\nu}$ is not the only propagating quantity. There are also perturbations of the connection:

$$\delta K^{ab}{}_\mu \neq 0,$$

which can propagate independently.

The ternary architecture predicts a new longitudinal gravitational mode h_L, induced by

$$\delta K \sim \partial J.$$

This mode satisfies a distinct wave equation:

$$h_L'' + \left(k^2 + \Delta_L(J, \partial J)\right) h_L = 0.$$

The effects of the longitudinal mode include:

- an additional component in the gravitational-wave signal,
- a slight time delay relative to the transverse modes,
- amplification or attenuation depending on $\dot{J}(t)$.

12.4 Gravitational birefringence

The term $\Xi(J)$ in Eq. (21) produces small differences between the velocities of the transverse modes:

$$v_+ \neq v_\times.$$

Birefringence arises from different contractions of internal phases:

$$\Xi_{+\times} \sim \mathrm{Tr}(P_1(\nabla J)P_2)\,,$$

$$\Xi_{\times+} \sim \mathrm{Tr}(P_2(\nabla J)P_1)\,.$$

This is a clear and unique prediction of the ternary architecture:

$$|\Delta v| = |v_+ - v_\times| \sim 10^{-18} - 10^{-16}.$$

Although extremely small, this difference is in principle detectable by:

- LISA (low frequencies),
- DECIGO and Einstein Telescope (high frequencies),
- polarization-resolved GW–EM correlations.

12.5 Dispersion of gravitational waves

The ternary architecture also introduces a dispersive contribution

$$\omega(k) = k\,\left(1 + \eta(J, \nabla J)\frac{k^2}{M^2}\right),$$

an effect absent in GR.

The predicted time delay is

$$\Delta t_{\mathrm{disp}} \sim 10^{-3} - 10^{-1}\ \mathrm{s}$$

for events such as:

- massive BH–BH mergers observable with LISA,
- NS–NS mergers observable with ET/CE,
- GW events with electromagnetic counterparts.

12.6 Internal interference in gravitational waves

Because of the ternary internal structure, gravitational waves can carry an internal phase

$$\Phi_{\mathrm{GW}} \sim \mathrm{Tr}(J\dot{J}),$$

which produces:

- internal rotation of polarizations,
- "internal precession"-type effects,
- signatures in amplitude modulation.

These effects are detectable only with next-generation instruments.

12.7 Three-point prediction

The ternary architecture simultaneously predicts:

- a longitudinal gravitational mode (h_L),
- gravitational birefringence ($v_+ \neq v_\times$),
- J-dependent gravitational dispersion.

The absence of any one of these three signatures rules out the model.

12.8 Conclusion of the section

The additional gravitational modes and gravitational birefringence are direct consequences of ternary torsion and of the internal variation of J. These effects:

- are absent in general relativity,
- are robust against details of the internal potential,
- fall within the projected sensitivity of future experiments,
- represent some of the clearest tests of the ternary architecture.

In Section XIII, we extend the analysis to the cosmological, astrophysical, and observational implications of these ternary gravitational modes.

13 Astrophysical and cosmological implications of ternary gravitational modes

The predictions of the ternary architecture are not limited to modified gravitational-wave propagation; they also have significant consequences for astrophysics, cosmology, large-scale structure, black-hole dynamics, compact-star evolution, and multimessenger correlations. In this section we analyse these implications in detail, both in weak-field regimes (linear cosmology) and in extreme regimes (black holes, neutron stars, supernovae).

13.1 Gravitational modes in strong-field regimes

In the presence of ternary torsion, strong gravitational fields—black holes, neutron stars, compact mergers—amplify the effects of the contorsion $K^{ab}(J)$.

Gravitational perturbations take the form

$$h_{\mu\nu} = h_+ + h_\times + h_L, \qquad h_L \propto (\nabla J).$$

Near extreme fields:

- the amplitude of the longitudinal mode increases,
- gravitational birefringence is enhanced,
- gravitational dispersion can accumulate over long distances,
- the internal phases may be modulated by the local gravitational potential.

This is particularly relevant for LIGO/Virgo/KAGRA observations and for the future Einstein Telescope.

13.2 Black holes and ternary geometry

Ternary torsion can subtly modify the structure of the region near the event horizon:

$$T^a \propto (\nabla J) \quad \Rightarrow \quad \delta\kappa_{\text{surf}} \neq 0,$$

where κ_{surf} is the surface gravity.

Possible consequences include:

- small deviations in quasi-normal mode (QNM) frequencies,
- modifications of the ringdown phase,
- detectable differences in damping times,
- signatures in post-merger gravitational-wave signals.

The LISA mission will be particularly sensitive to such deviations.

13.3 Neutron stars and dense matter

Ternary torsion introduces an axial coupling into the Dirac equation, which can affect:

- degenerate pressure,
- chemical equilibrium in compact stars,
- phase transitions in dense matter,
- the effective elastic modulus of neutron-star crusts.

These effects can lead to

$$\Delta R_{\text{NS}} \sim 10^{-2} - 10^{-1}\ \text{km},$$

in the radius of a neutron star—an interval detectable via NICER and through gravitational-wave pattern measurements from NS–NS mergers.

13.4 Supernovae, neutrinos, and ternary torsion

Ternary torsion modifies the Dirac equations for neutrinos:

$$\nu_i \longrightarrow e^{i\Phi_i(J,\nabla J)}\nu_i.$$

In supernovae:

- internal phases can affect MSW transformations,
- neutrino fluxes can exhibit subtle energy variations,
- neutrino oscillations may follow ternary symmetry–modified patterns,
- timing and synchronization of the fluxes may show observable shifts.

These effects are, in principle, testable with DUNE and Hyper–Kamiokande.

13.5 Large-scale cosmological implications

Ternary cosmology yields dynamic dark energy:

$$\rho_{\text{eff}}(z) \sim \text{Tr}(J\dot{J}),$$

which implies

$$w(z) = -1 + \epsilon(z), \qquad |\epsilon| \sim 10^{-2}.$$

This behaviour produces:

- deviations in the distance–redshift relation,
- modifications in $H(z)$ that can alleviate the H_0 tension,
- cumulative effects on structure formation,
- differences in the large-scale correlation function.

DESI-II, Euclid, and the Roman Telescope will be able to test these predictions.

13.6 Anisotropies and directional signatures

If $\nabla_i J \neq 0$ on cosmological scales, then

$$\frac{\nabla_i \alpha}{\alpha} \neq 0, \qquad v_+ \neq v_\times \quad \Rightarrow \quad \text{directional anisotropies in GW.}$$

This would produce:

- a dipole in $\Delta\alpha/\alpha$,
- directional effects in GW dispersion,
- direction-dependent polarization in gravitational lensing,
- combined GW+quasar absorption signatures.

13.7 Multimessenger interdependence

The most significant conceptual result of this section is the multimessenger coherence.

The dynamics of J simultaneously produces signatures in

$$\begin{cases} \text{gravitational waves,} \\ \text{gravitational lensing,} \\ \text{electromagnetic spectra of quasars,} \\ \text{neutrino oscillations,} \\ \text{dark energy,} \\ \text{large-scale structure.} \end{cases}$$

This interdependence is unique to the ternary architecture.

13.8 Conclusion of the section

The additional gravitational modes, ternary torsion, and the internal dynamics of J simultaneously affect the behaviour of black holes, neutron stars, supernovae, neutrino fluxes, and cosmic expansion. The predictions are:

- strongly correlated,
- directly observable in multiple channels,
- independent of the detailed form of the internal potential,
- accessible to current and forthcoming instruments.

The next section (XIV) synthesizes the entire architecture into a unified conceptual perspective and defines explicit falsifiability criteria for the theory.

14 Conceptual synthesis and falsifiability criteria

The ternary architecture proposed in this work is not an incremental extension of the standard paradigm, but a conceptual reconstruction of how geometry, internal structure, and fundamental fields interact. The identity $J^3 = -I$, together with ternary projections and the possibility of geometric torsion, yields a structural unification between phenomena that, in the standard theory, appear as unrelated: variation of fundamental constants, cancellation of vacuum energy, neutrino oscillations, and gravitational-wave propagation.

In this section we synthesize the architecture at the conceptual level and formulate explicit falsifiability criteria, so that the model can be rigorously tested by future experiments.

14.1 Conceptual synthesis

The fundamental element of the architecture is the internal ternary operator J, an object with discrete properties but continuous dynamics. The cubic relation $J^3 = -I$ generates three internal projections P_0, P_1, P_2, which decompose every fundamental field into three coherent components. This internal tripartition underlies:

- neutrino oscillations in three coherent modes,
- ternary interference that cancels the vacuum energy,
- variation of the gauge normalization and of the fine-structure constant,
- the appearance of geometric contorsion,
- additional gravitational modes and birefringence.

The link between internal structure and geometry is mediated by the compatibility condition

$$\nabla_\mu P_k = 0,$$

which translates into the cubic identity

$$J^2(\nabla_\mu J) + J(\nabla_\mu J)J + (\nabla_\mu J)J^2 = 0.$$

This identity makes a purely Levi–Civita connection impossible and enforces torsion. Thus, torsion is not "optional" or merely "phenomenological", but the mathematically inevitable outcome of ternary symmetry.

14.2 Central role of the dynamics of J

The dynamics of J—namely the temporal evolution or small spatial variations of the internal operator—is the source of all deviations from the standard paradigm. This dynamics directly produces

$$\mathrm{Tr}(J\dot{J}),$$

the source term that appears simultaneously in:

- dynamic dark energy,
- the variation of $\alpha(z)$,
- neutrino phases,
- contorsion and additional gravitational modes.

This uniqueness in predictions is essential: instead of a large number of free parameters, the ternary architecture reduces everything to a single internal object whose dynamics drives the full set of effects.

14.3 Phenomenological unification

The model unifies five domains of physics within a single structure:

$$(\text{gravity},\ \text{vacuum energy},\ \text{fundamental constants},\ \text{neutrinos},\ \text{cosmology}) \quad \longleftrightarrow \quad J(x).$$

This unification is structural, not imposed phenomenologically.

14.4 Simplicity of the principles

The ternary architecture rests on only three fundamental ideas:

1. the internal identity $J^3 = -I$;
2. Riemann–Cartan geometry (vielbein + connection + torsion);
3. projective compatibility $\nabla_\mu P_k = 0$.

All cosmological, gravitational, and particle-physics predictions follow from these three ingredients.

14.5 Falsifiability criteria

To be scientific, a theory must be refutable by observation. The ternary architecture is remarkably falsifiable in a multiple and simultaneous way. The model is ruled out if any of the following experimental conditions fails.

1. Variation of $\alpha(z)$ If future measurements (ELT–HIRES, ESPRESSO, SKA) find that

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 10^{-9} \quad \text{for } 0 < z < 5,$$

then the ternary architecture is excluded.

2. Absence of the longitudinal gravitational mode If LISA, DECIGO, or the Einstein Telescope definitively confirm that

$$h_L = 0$$

for all relevant sources, the model is excluded.

3. Absence of gravitational birefringence If it is established that the propagation speeds of the two GW modes are identical,

$$v_+ = v_\times,$$

with precision better than 10^{-18}, the theory is ruled out.

4. Absolute constancy of $w(z)$ If observations confirm that

$$w(z) = -1 \quad \text{with precision} < 10^{-2} \quad \text{for } 0 < z < 3,$$

then dark energy is not dynamic and the model is excluded.

5. Neutrino sector: CP phase If DUNE and Hyper–Kamiokande measure

$$\delta_{\mathrm{CP}} \notin (120^\circ \pm 10^\circ),$$

the model is excluded.

6. Absence of mixing-variation in the neutrino sector If it is experimentally established that

$$\partial_E \theta_{ij} = 0 \quad \text{and} \quad \partial_t \theta_{ij} = 0,$$

at levels below 10^{-3}, the model is excluded.

14.6 Strength of the criteria

What makes the ternary architecture strongly scientific is that it does not require ad hoc adjustments: *either all of its predictions hold, or the theory is rejected as a whole.*

This level of rigidity is rare among beyond–standard models.

14.7 Conclusion of the section

The ternary architecture proposes a unified and unusually coherent perspective on nature: a single internal object J governs gravity, neutrino oscillations, the dynamics of fundamental constants, and the cancellation of vacuum energy. The model is highly falsifiable, predictive, and firmly grounded in modern differential geometry.

In Section XV, we present the general conclusions of the work and outline future directions for testing.

15 Conclusions and future directions for testing

The ternary architecture proposed in this work offers a conceptual reconstruction of the relationship between geometry, internal structures, and fundamental dynamics. By introducing an internal operator J satisfying the cubic identity $J^3 = -I$, together with unavoidable geometric torsion and the associated ternary projections, the theory provides a unified framework in which:

- vacuum energy is structurally cancelled;
- the fine-structure constant α varies in time in a controlled and predictable way;
- neutrino oscillations and a geometric CP phase derive from the same internal structure;
- gravitational-wave propagation is modified through the appearance of a longitudinal mode and birefringence;
- dark energy becomes an internal dynamical effect rather than a fundamental constant;
- torsion cosmology offers new perspectives on current observational tensions.

Each of these results is a direct consequence of ternary symmetry and of the compatibility condition $\nabla_\mu P_k = 0$, not an additional hypothesis. The theory does not rely on arbitrarily tunable parameters, which makes it highly predictive and strongly falsifiable.

15.1 Conceptual impact

The proposed model introduces a paradigm in which internal dynamics and external geometry are not separate entities, but two facets of the same mathematical object. The link between the internal ternary operator and the geometric connection leads to a new view of gravity: internal structure determines contorsion, and contorsion modifies the propagation of gravitational information, of particles, and of variations in fundamental constants.

This unified perspective is both conceptually elegant and mathematically rigorous, preserving all standard geometrical symmetries while extending them in a natural way.

15.2 Testability and falsifiable predictions

The ternary architecture is one of the few beyond–standard theories that simultaneously yields predictions that are:

- quantitatively specified,
- cross-checkable across multiple observational channels,
- robust against parametric variations,
- directly falsifiable by experiment.

Major experimental tests include:

- measuring the variation of $\alpha(z)$ with 10^{-9} precision (ESPRESSO, ELT-HIRES);
- detecting or ruling out the longitudinal gravitational mode (h_L);
- determining gravitational birefringence at the 10^{-18} level (LISA, DECIGO, ET);
- measuring the CP phase in the neutrino sector (DUNE, Hyper–Kamiokande);
- determining the behaviour of $w(z)$ with sub-10^{-2} precision (Euclid, Roman Telescope);
- probing extremely small deviations in neutrino mixing angles ($\partial_E \theta_{ij}$, $\partial_t \theta_{ij}$).

Confirmation of any one of these signatures reinforces the entire architecture. Their absence directly excludes it.

15.3 Future directions

Several theoretical and phenomenological directions merit further exploration:

1. Full analysis of torsion instabilities. A detailed treatment of contorsion modes at the nonlinear level may clarify possible instabilities or additional emergent phenomena.

2. Coupling the ternary architecture to GUT or supersymmetric models. This could embed ternary symmetry in a broader framework, allowing a classification of particles according to internal phases.

3. Connection with noncommutative spaces. The ternary operator can be viewed as a particular form of internal cyclicity, with intriguing analogies to noncommutative geometries.

4. Extended analysis of the neutrino sector. A detailed study of the correlation between neutrino oscillations and torsion dynamics may yield sharper predictions for future experiments.

5. Nonlinear analysis of ternary gravitational waves. Numerical simulations of BH–BH mergers in this framework may reveal new gravitational structures.

6. Precise estimates of cosmological deviations. A full formulation of FRW equations with ternary torsion could generate detailed predictions for $H(z)$ and the galaxy distribution.

15.4 Closing remarks

The ternary architecture represents a structural unification proposal that profoundly connects: gravity, the fundamental constants, vacuum energy, neutrinos, cosmology and atomic nuclei.

Through its mathematical simplicity and predictive power, the theory offers a coherent perspective on some of the most persistent problems of modern physics, including both the dynamics of the vacuum and of the fundamental interactions, as well as the structure and stability of atomic nuclei.

Regardless of whether its predictions will be confirmed or refuted, the ternary architecture opens a new conceptual path in the exploration of the fundamental interactions, the physical vacuum, and nuclear matter.

Appendix A: Additional mathematical details

A.1 The cubic identity and the ternary projective space

For any internal operator J satisfying $J^3 = -I$, the minimal polynomial is

$$p(\lambda) = \lambda^3 + 1,$$

with roots

$$\lambda_k = e^{i(\pi+2\pi k)/3}, \qquad k = 0, 1, 2.$$

The associated spectral projectors are

$$P_k = \prod_{j \neq k} \frac{J - \lambda_j I}{\lambda_k - \lambda_j}, \qquad k = 0, 1, 2.$$

They satisfy

$$P_k^2 = P_k, \qquad P_k P_j = 0 \ (k \neq j), \qquad \sum_{k=0}^{2} P_k = I.$$

Geometrically, the triplet $\{\lambda_0, \lambda_1, \lambda_2\}$ forms an equilateral triangle on the unit circle, with vanishing sum, $\lambda_0 + \lambda_1 + \lambda_2 = 0$, in direct analogy with the identity $1 + \omega + \omega^2 = 0$ for the cubic roots of unity.

A.2 Ternary contorsion

For the total connection

$$\omega^{ab} = \omega^{ab}_{\mathrm{LC}} + K^{ab}(J),$$

the contorsion is given by

$$K_{abc} = \frac{1}{2}(T_{abc} - T_{bca} + T_{cab}).$$

The structural relation $T^a \propto (\nabla J) \cdot e^a$ follows from the condition $\nabla_\mu P_k = 0$.

A.3 Internal equation for J

The full variational equation reads

$$\nabla_\mu(\partial^\mu J) - \frac{\partial \mathcal{V}}{\partial J} + \mathcal{C}[J, \omega] + \mathcal{I}[J, P_k] = 0.$$

The terms $\mathcal{C}$ and $\mathcal{I}$ are fixed by the cubic identity.

A.4 Ternary neutrino oscillations

For

$$\nu = \nu_0 + \nu_1 + \nu_2, \qquad \nu_k = P_k \nu,$$

the ternary Dirac equation yields the system

$$(i \not{\nabla} - m_k)\nu_k + \sum_j P_k(\nabla J) P_j \nu_j = 0.$$

—

Appendix B: Derivation of the longitudinal gravitational mode

Starting from the perturbations

$$\delta e^a{}_\mu, \qquad \delta\omega^{ab}{}_\mu,$$

with

$$\delta\omega^{ab} = \delta\omega^{ab}_{\mathrm{LC}} + \delta K^{ab}(J),$$

one obtains the generalized wave equation

$$h''_{ij} + k^2 h_{ij} + \varepsilon(J, \partial J) h_{ij} = 0.$$

The term ε arises from the variation of the contorsion:

$$\delta K^{ab} \sim \partial J.$$

The longitudinal projection h_L satisfies a separate equation,

$$h''_L + (k^2 + \Delta_L) h_L = 0.$$

—

Appendix C: Relation between $\alpha(z)$ variation and vacuum energy

For the gauge normalization

$$\alpha_{\mathrm{eff}} = \alpha_0 / Z(J),$$

with

$$Z(J) = \mathrm{Tr}(P_0 + \omega P_1 + \omega^2 P_2),$$

we obtain

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta Z}{Z} = -\frac{\mathrm{Tr}[(\nabla J)\Xi(J)]}{Z}.$$

The term $\mathrm{Tr}(J\dot{J})$ appears simultaneously in

$$\rho_{\mathrm{eff}}(z), \qquad \frac{\Delta\alpha}{\alpha}, \qquad \Phi_\nu, \qquad \text{GW propagation.}$$

—

Appendix D: FRW equations with ternary torsion

In a homogeneous universe,

$$T^i{}_{0j} = \tau(t)\delta^i{}_j, \qquad \tau(t) \propto \dot{J}(t).$$

The Friedmann equation is modified as

$$H^2 = \frac{\kappa}{3}\rho_{\mathrm{eff}} + \Delta H_{\mathrm{tors}}, \qquad \Delta H_{\mathrm{tors}} \sim \mathrm{Tr}(J\dot{J}).$$

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